MATHEMATICAL MODEL SIMULATION OF CANAL SYSTEMS AND CONTROL ALGORITHMS

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by
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INTRODUCTION

The operation of an open channel conveyance system is very complex. The capabilities, limitations, and the operating characteristics need to be known if the goal of achieving optimum response and efficiency is to be accomplished when certain quantities of water are to be conveyed downstream and delivered through canalside turnouts at specified times.

Open channel conveyance systems, usually trapezoidal in cross section, delivering water to irrigation project lands downstream, consist of a series of canal reaches or pools separated by check gate structures. Check gate structures maintain water surface elevation upstream at desired levels and regulate the flow into the downstream canal reach as deliveries to canalside turnouts are made en route. Canal check structures also assist in the rapid mass transfer of water throughout the canal system. The canal reach between check gate structures usually includes intermediate structures such as siphons, transitions, in-line reservoirs, pumping plants, and many canalside turnouts. Spillways or wasteways are sometimes located near natural cross-drainage channels and are used to discharge excess water that may occur in the operation of the canal system. Some canal systems have drainage water or other sources of water supply discharging into the canal prism at several locations en route, all of which have an influence on the discharge rates along the canal system. Other factors such as the type of canal lining used (compacted earth, concrete, or natural conditions), the length of the canal reaches, obstructions (bridge piers, bends, etc.), and the cross-section properties all contribute to hydraulic losses in the system and influence the surge wave (by which flow changes propagate downstream) amplitude and travel times, directly affecting the operation of the canal system.

It is not necessary to wait until the canal system is in operation to gain operational experience, i.e., to define the capabilities, limitations, and operating characteristics after the canal is constructed and then put into operation.

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Mathematical modeling of canal systems has been developed and operational for many years. A mathematical model can simulate a canal

system with reasonable accuracy and should be used extensively during early stages of design to identify the operational characteristics. In the early stages of design, mathematical modeling can identify the conveyance system capabilities and limitations and assist in the location of check gate structures and/or regulating reservoirs to provide optimum operational efficiency. Mathematical modeling also is very helpful and in some cases necessary in the selection of optimum control schemes to provide automatic flow regulation of the canal system.

Therefore, the necessary operating experience including knowledge of the selected control system response characteristics can be gained "on-the-bench" using mathematical models. It is desirable, however, to verify the mathematical model results after the canal and control system are in operation to be sure the mathematical model has simulated the canal system correctly. Field tests for verification do not need to be extensive and costs would be minimal.

The mathematical model for a digital computer consists of a program of equations and logic statements for all the elements involved in the process of simulating an actual operating canal system and the flow regulating control algorithms. The basic fundamentals of each element involved are described as opposed to a methodical discussion of the computer program statements. A brief description of each element of the mathematical model is first presented. Each element is then described in greater detail including comments on problems or limitations that may be encountered in the element. Finally, a summary and conclusions are offered.

MATHEMATICAL MODEL ELEMENTS

General

The canal and control system mathematical model has three basic phases: (1) initialization, (2) operation, and (3) output. The initialization phase begins the canal and control system at a specified steady-state flow condition. The operation phase executes the desired change of flow condition and solves the program equations at successive time intervals for a specified period of time. The output phase presents the results of the operation in a format suitable to allow easy review and analysis of the canal and control system response characteristics. Each phase of the mathematical model has elements which constitute basic activities in the process of initializing, operating, and observing the output of a simulated actual canal system.

Brief Description of Each Element

INITIALIZATION

INPUT	Canal physical properties, boundary conditions, and control system control parameters.
STATIONS	Determines location of computation points subject to the requirement they be at least a specified distance, DX, apart.
BACKWATER	Establishes a backwater profile to initialize depth, Y, discharge, Q, and velocity V, all stations.
INIT	Calculates the initial values of celerity, C, initial gate opening, G, and the characteristic lines of the unsteady flow equations for all stations.

OPERATION

TIME STEP	Computes the successive time intervals, DT, subject to the requirement that DX/DT must be greater than (C+V)/0.98 to give minimum interpolation errors.
MANOP	Imposes desired conditions, water levels upstream and/or downstream and check gate openings at boundaries. Manual operation of the canal system can be simulated in this element.
METHOD OF CHARACTERISTICS	Solves the unsteady flow equations for each canal reach.
GBEG GMID GEND	Computes the new Y and Q at all the boundaries such as check gates, siphons, reservoirs, turnouts, canal transition, etc.

SETGATE

CONTROL ALGORITHM

Simulates the prototype control system algorithms which determine the new gate position, usually based on current values of Y.

CELERITY

Calculates the new wave celerity at all stations.

STORE

Stores all desired values Y, Q, G, etc., needed for the output phase time history plots at selected stations.

TIME

The operation phase is repeated until the successive time intervals, DT, accumulate to the specified period of time which ends the simulation process.

OUTPUT

STORE

Recalls all the desired values, Y, Q, G, etc., stored during the operation phase at the selected stations.

OUTPUT

Presents the results of the operational phase in a format suitable for review and analysis of the response characteristics of the simulated actual canal and control system at selected stations. The usual output format is a time history plot of the Y,Q, and G at all the check gate boundaries.

Detailed Description of Each Element

INPUT

In order to simulate an actual operating canal and control system all of the canal physical properties, the boundary conditions, and control parameters must be read into the program. A list of the input values are as follows:

Geometry	Boundary	Boundary conditions
Bottom width	Number of boundaries	Initial discharge, Q
Side slope	Boundary type	
Bottom slope	Canal invert change	Initial water level, Y
Length	Siphon invert change	Initial gate opening, G
Gate width	Siphon coefficient	Change of turnout discharge (initial and future)
	Canal friction factor	Stations to be plotted
	Distance interval, DX	Time (period of simulation)
	Gate coefficients of discharge	

All the input values are organized and read into the program for each boundary and represent conditions upstream. Each boundary is given an identification number, 1 for check gates, 2 for siphons, etc. The length is the accumulated distance to each boundary starting at the beginning of the canal system and does not include the length of the siphons. The siphon coefficient when multiplied by the discharge, Q, squared represents the head loss for the siphon. The boundary conditions establish the initial steady-state flow condition at time zero. The initial and future canalside turnouts are also established. The future canalside turnout changes would represent a preplanned operational scheme to test the canal and control system response characteristics. If the actual canal system has additional sources of water flowing into the canal prism at locations en route, the canalside turnout boundary can be used for this purpose using a negative discharge value. All the other input values are self-explanatory.

STATIONS

Using the input values of the accumulated length and specified distance, DX, for points of computations, each canal reach between boundaries is divided into subreaches. Each subreach is subject to the requirement they be at least DX apart. Each DX is then given a station number, station 1 being the head of the canal system. The calculation of stations continues for each canal reach to the end of the canal or to the last boundary.

BACKWATER

The depth, Y, discharge, Q, and velocity, V, are initialized for all stations by establishing a backwater profile for each canal reach between boundaries. Since the length, DX, of each subreach and the depth, Y, at one end (the initial input depth upstream from each boundary) are known, the depth at the other end (the next upstream station) can be determined by successive approximations. Once the depth, Y, is calculated and since steady-state discharge, Q, and cross section geometry is read into the program, the velocity, V, can then be calculated for all stations, including the cross sectional area, A, and the water surface top width, T.

INIT

The initial values of celerity, C, for each station are calculated by the equation $C = \sqrt{gA/T}$ where g is the acceleration of gravity, A is the cross section area, and T is the top width of the water surface (A and T are determined in the backwater element above). The initial gate opening is also calculated based on the orifice equation $G = Q/GW{*}CD{*}\sqrt{2g{*}\Delta H}$ where Q is the initial discharge, GW is the gate width, CD is the coefficient of discharge, and ΔH is the head differential across the check gate structure. The head differential, ΔH, includes adjustments for a siphon usually located immediately below the check gate and changes of canal invert across the gate boundary. The coefficient of discharge is usually read into the program as a table of values of Y/G versus CD. Therefore, the initial gate opening is used to determine an estimated CD for the orifice equation. The computed initial gate opening is arrived at by an iterative procedure. The initial values for use in the method of characteristics are calculated for all stations.

TIME
STEP

The first element of the operation phase is to choose a time interval, DT, so that in all subreaches $DT < DX/(V \pm C)$. The procedure is to first search all $(V \pm C)$ values to find the maximum value. Since DX is known, the value of the successive time interval, DT, can be determined and is subject to the requirement that the ratio DX/DT is greater than $(V+C)/0.98$ to give the minimum interpolation errors of the method of characteristics discussed in subsequent paragraphs. The procedure described for calculating the best time interval, DT, is satisfactory if the geometry and flow conditions do not vary significantly throughout the length of the canal system. The length of subreaches, DX, is usually selected so that the successive time intervals, DT, are about 60 seconds. If the canal geometry and flow conditions vary significantly, then either additional programming is required to allow for a variable time interval, DT, suited for each canal reach or larger interpolation errors will have to be tolerated.

MANOP

This element in the operational phase is used to impose desired conditions at a boundary at specified times. Water levels upstream and/or downstream from a boundary can be fixed or changed at specified times. Gate openings can also be fixed into a desired position and changed at specified times permitting a simulation of a manual operational mode of control. This element is often used to impose a perturbation on the flow conditions of the canal by changing the most upstream and/or downstream check gate position.

METHOD OF CHARACTERISTICS

Wylie and Streeter [1]*, among others, have shown that the continuity and momentum equation for flow in an open channel form a pair of hyperbolic partial differential equations. These can be expressed in the form of four total differential equations which are known as the characteristic equations. Several different numerical methods can be used to solve the characteristic equations.

The numerical method used to solve the open channel flow problems described in this paper employs a fixed time and space grid. This method has the advantage that conditions at boundaries can be determined at fixed times. The method also uses interpolations along the distance line to determine values that are used to extend the grid in time. Because of the many interpolations the solution does not approach a steady-state condition. Instead, the canal appears to be "leaky."

Even though the solution will not approach a steady-state condition, it has predicted observed transients on a prototype canal with a reasonable degree of accuracy, Buyalski [2].

GBEG
GMID
GEND

The principal types of boundaries that are found in canal systems are check gates, siphons, reservoirs, turnouts, transitions, and pumps. At interior boundaries, which separate canal segments, an energy equation is used to relate flow depths and discharges across the boundary. At terminal boundaries, which occur at the ends of an aqueduct, it is necessary to specify either depth or discharge schedules. Normally, check gates, siphons, turnouts, and transitions comprise interior boundaries; whereas, pumps and reservoirs comprise terminal boundaries. The specific type of boundary and its associated functional relationship is determined by the appropriate values in the input phase of the initialization segment of the program.

SETGATE

CONTROL SYSTEM

The SETGATE element provides an interface between simulated canal system and the selected control system. The new gate position for the next successive time interval is determined from the control system by means of algorithms. It is emphasized

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* Numbers in brackets refer to literature cited.

the algorithms must exactly duplicate the prototype control system including logic and identical sources of input information. The control algorithms generally use current values of water depth, Y, and in some cases values of discharge, Q. The boundary or check gate being controlled and the station providing input to the algorithm are specified or written as part of the interface program. An example of a prototype control system algorithm is the proportional plus proportional reset mode of control [3] calculation of the gate opening $G = K1(YT-YF)+K2\int_0^t (YT-YF)dt$. K1 and K2 are proportionality factors or "gains," YT is a referenced target water level, and YF is a modified canal water level determined by its own algorithm using a current selected water level as a source of input information. The values of K1, K2, and YT are examples of control parameters and are provided in the INPUT element in the initialization phase of the mathematical model. Logic may also be included to compare the existing gate opening to the control algorithm new computed gate opening. The gate is then moved into the new position following a predetermined procedure consisting of a series of logic statements.

The control system is modeled by writing a program of equations or algorithms for all the elements of the selected control scheme. There are numerous control systems available for automatic flow regulation of canal check gates. The various control systems are not discussed. However, the SETGATE element provides a very convenient location in the mathematical model to simulate the selected prototype control system. Programs for the various control systems can be written and inserted into the SETGATE element when needed without affecting the program for simulating the canal system being controlled. The control system control parameters can be easily changed in the INPUT element. The mathematical model can be executed many times by a high-speed digital computer with large memory capacities [60,000 (octal) words minimum] in search for optimal control system design.

CELERITY

The new wave celerity $C = \sqrt{gA/T}$ is now calculated at all stations and will be used in the next successive time interval of the operational phase.

STORE

All of the desired values needed for the output phase time history plots are stored in a separate file at each time interval. Writing the information needed to a separate file at each time interval of computation reduces the central memory requirements of the computer program. Usually the values of water level, upstream and downstream; the discharge; and gate opening are written to the file for each boundary and are identified by the boundary number and the accumulated time.

TIME The accumulated time intervals, DT, are compared with the time period which ends the simulation process. If the accumulated time is less than the end time, the operational phase is repeated beginning with the TIME STEP element. If the accumulated time exceeds the end time, the operational phase is completed and the OUTPUT phase begins.

STORE The STORE element is the first activity of the OUTPUT phase and is identical to the STORE element of the OPERATIONAL phase. However, the information stored in the operational phase usually requires reorganization into a suitable format for printing or plotting the desired information needed to analyze the time history response characteristics of the simulated canal and control system. The reorganization consists of reading the STORE file for all the desired values of boundary No. 1 starting at time 0 and ending at the end of the time period writing the information to a second file in the process. The first file is rewound and the process begins again for boundary No. 2, etc. The end result is that all the data are arranged in a desired sequential order of boundary location and an increasing time history.

OUTPUT The final activity of the mathematical model is to output the desired information of the operational phase in a suitable format such that the response characteristics of the simulated canal and control system can be efficiently reviewed and analyzed. The data can be printed in numerical form and are used to obtain precise information. Usually the printed data are difficult to examine on an overall basis. An X-Y plotter or CRT (cathode ray tube) can present the data in graphical form, which provides the most convenient output for reviewing the time history of events. The X-Y plotters, however, are usually too slow. A microfilm of the CRT plot also is a slow process. The best peripheral device is a graphics CRT terminal which plots the data at high speeds using available software to execute a plotting program. Appendix I is an example of a graphic CRT terminal output consisting of a graph for each check gate boundary.

Appendix I illustrates the output phase of a simulated canal and control system that is 113 km (70 miles) long having 12 check structures. The time history plot includes the response characteristics of the upstream water level, the gate opening, and the discharge for each check gate structure for a 24-hour period of time. The control system controlling the canal check gates is the proportional plus proportional reset mode of control for automatic downstream control.

The mathematical model computer program required 60,000 (octal) words of central memory to execute the initialization and operational phase. A second plotting program was used for the output phase requiring 120,000 (octal) words of central memory. Both programs took a total of 1,500 seconds of computer time to execute and about 30 minutes of connected time for the graphics CRT terminal to plot the time history graphs.

SUMMARY AND CONCLUSIONS

The basic elements of activity of a mathematical model which simulates the unsteady-state flow in a canal system, including the capability of a control system to regulate check gates, has been described. A computer program written for the model can be used to design, confirm, and/or assist in developing operational criteria and a control system for a particular canal. The purpose of the computer program is to allow analysis of an actual operating canal and control system to be conducted "on-the-bench."

The construction of a mathematical model is not difficult. The initialization phase describes the properties of the canal: the canal configuration, the cross section properties of each reach, the gate properties, and the control parameters. The initial steady-state depths and discharges above the gates and other boundaries are specified, as well as the times and magnitudes of subsequent discharge changes, and any conditions which are to be held constant at a boundary. The operational phase executes the desired operational scheme for a specified period of time. The numerical solution of the partial differential equations for continuity and momentum for the unsteady-state flow conditions is accomplished by the method of characteristics having a digital computer solving the equations at successive time intervals. At the boundaries, the inlet and outlet water levels and discharge are matched with the corresponding values at the beginning and end of adjoining prismatic channel subreaches by simultaneous solution of equations and by iterative procedures. The mathematical model provides a very convenient location in the operational phase to insert the selected prototype control system algorithms which are interfaced to the simulated canal system. The output phase provides a convenient graphical picture of the canal and control system response characteristics during the period of interest.

The mathematical model simulating a canal and control system can produce results which agree, within reasonable accuracy, with the actual behavior of the real open channel system. Therefore, the performance of a control system can be predicted with a great deal of confidence. The mathematical model requires a relatively large computer facility to execute the computer program and produce suitable output graphs. When using a digital computer facility, a period of time is required for a complete new solution to evaluate the effects of each change in flow and/or the control parameters. The period of time may be only minutes, and may be hours if the computer system is busy with other users, even on a large machine. These inconveniences are minor compared to the magnitude of benefits received by performing the study "on-the-bench." Performing similar studies on a real canal system or on a hydraulic laboratory model can be very expensive, time consuming, and often incomplete. However, mathematical modeling requires computer programming skills, a knowledge of open-channel hydraulics, and field operating experience is desirable when developing and implementing practicable control schemes.

LITERATURE CITED

[1] Wylie, E. B., and V. L. Streeter, "Fluid Transients," McGraw-Hill Book Co., New York, 1977

[2] Harder, J. A., M. J. Shand, and C. P. Buyalski, "Automatic Downstream Control of Canal Check Gates by the Hydraulic Filter Level Offset (HyFLO) Method," a paper presented at the Fifth Technical Conference, U.S. Committee on Irrigation, Drainage, and Flood Control, Denver, Colorado, October 8-9, 1971, and to the International Commission on Irrigation and Drainage, Eighth Congress, Varna, Bulgaria, May 1972

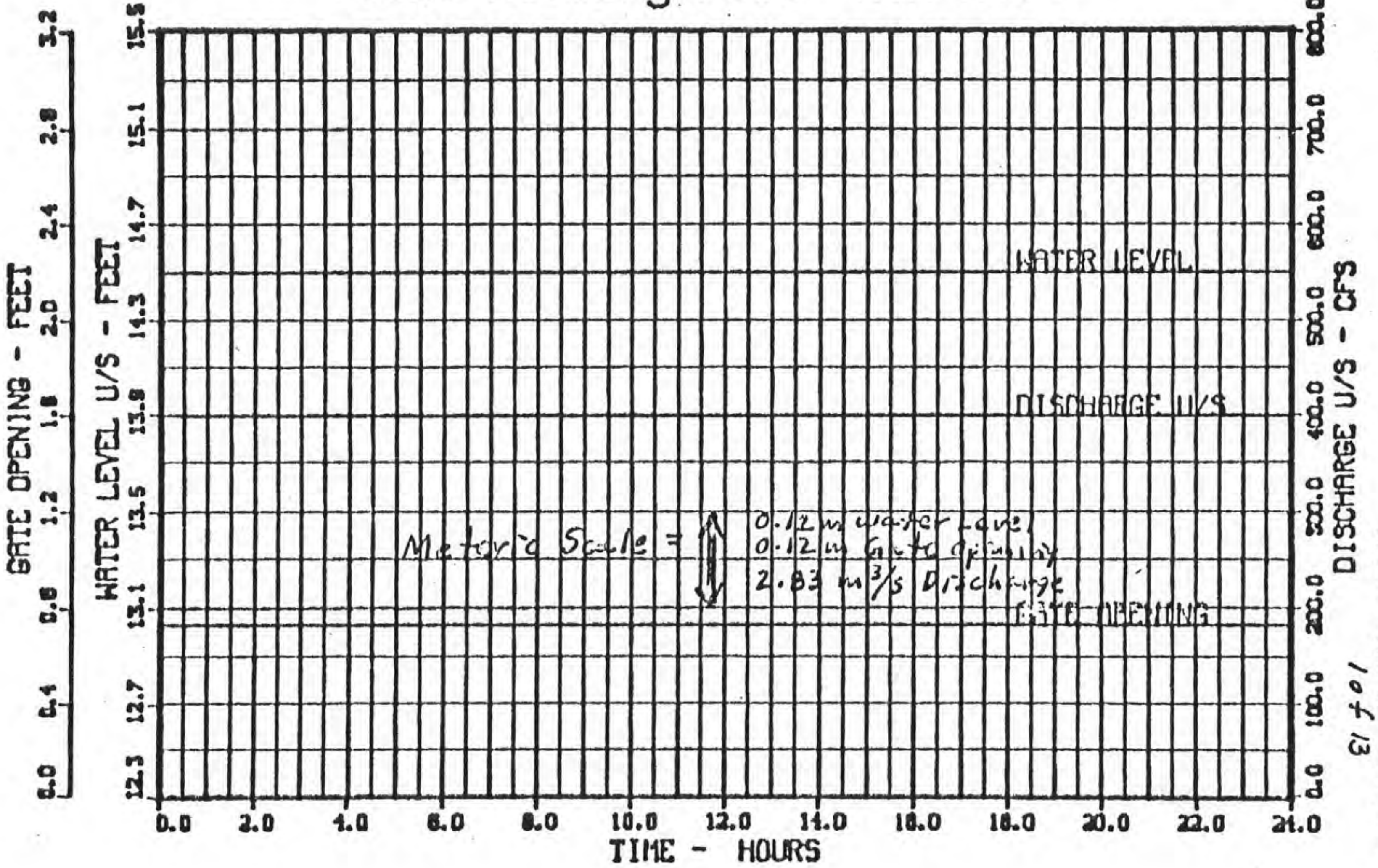
[3] Buyalski, C. P., and E. A. Serfozo, "Study of Electronic Filter Level Offset (EL-FLO) Plus RESET Equipment for Automatic Downstream Control of Canals," REC-ERC-79-3, Engineering and Research Center, U.S. Bureau of Reclamation, Denver, Colorado (in preparation)

APPENDIX I

A sample of a graphics CRT terminal output using the Navajo Indian Irrigation Project canal system controlled by the automatic downstream proportional plus proportional reset mode of control known as the EL-FLO plus RESET controller as an example. Each of the 12 check gate boundaries (including the end of the canal noted as check No. 13) has a time history plot for a 24-hour period of the water level upstream, the gate opening, and the discharge. Therefore, the response characteristics of the canal resulting from control action by the EL-FLO plus RESET controller, which automatically regulates the upstream check gate using the downstream water level, can be analyzed. The downstream water level input is located at the downstream end of the downstream canal reach or pool between two check gate structures. The graph vertical scales are in English units.

N.I.I.P. CANAL SYSTEM - CHECK NO. 1

Time History Plot Run No. 8

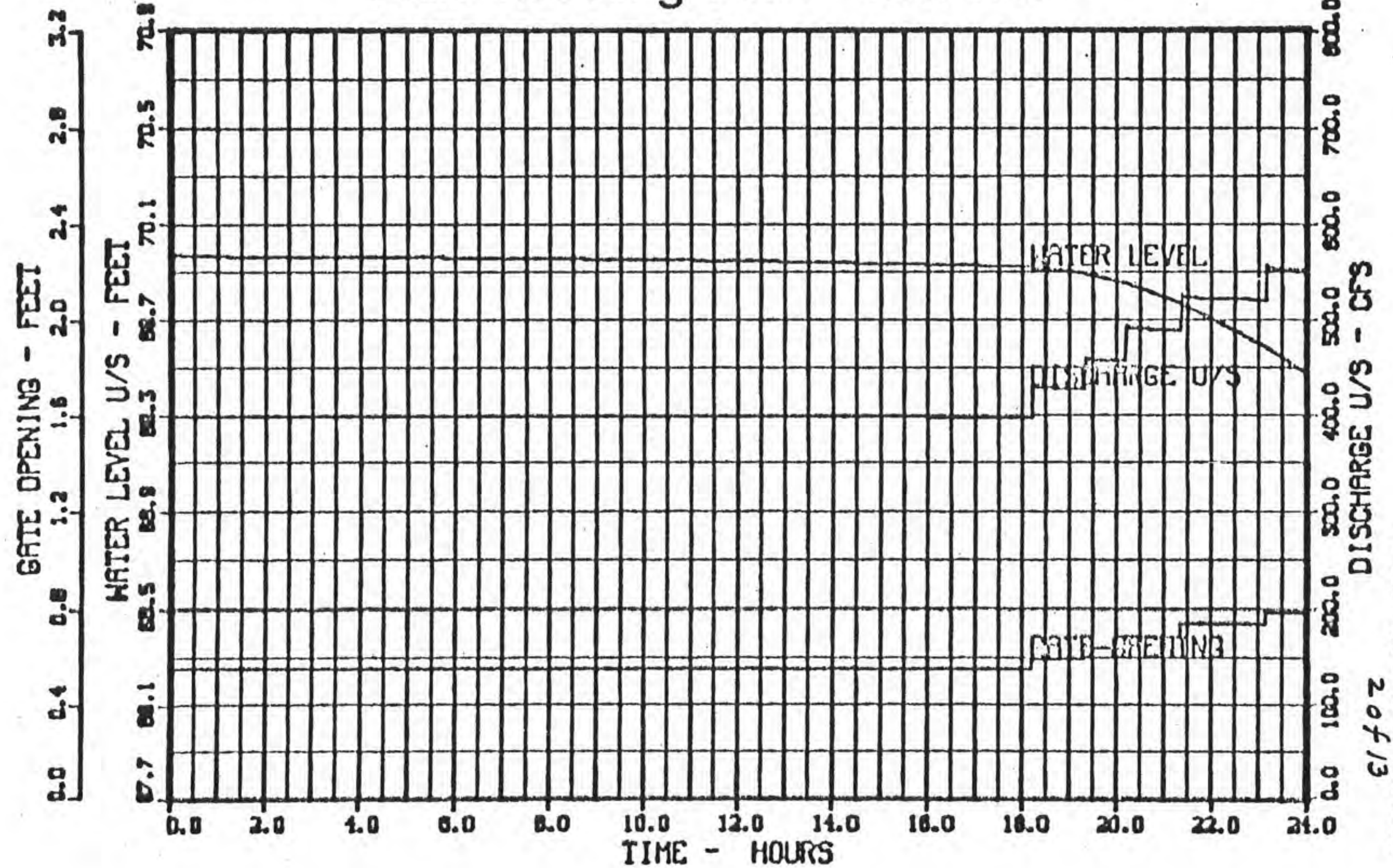


Metric Scale = ⇕ 0.12 m water level
0.12 m Gate opening
2.83 m^3/s Discharge

Automatic Downstream Control
1 of 13

N.I.I.P. CANAL SYSTEM - CHECK NO. 2

Time History Plot Run No. 8



Automatic Downstream Control
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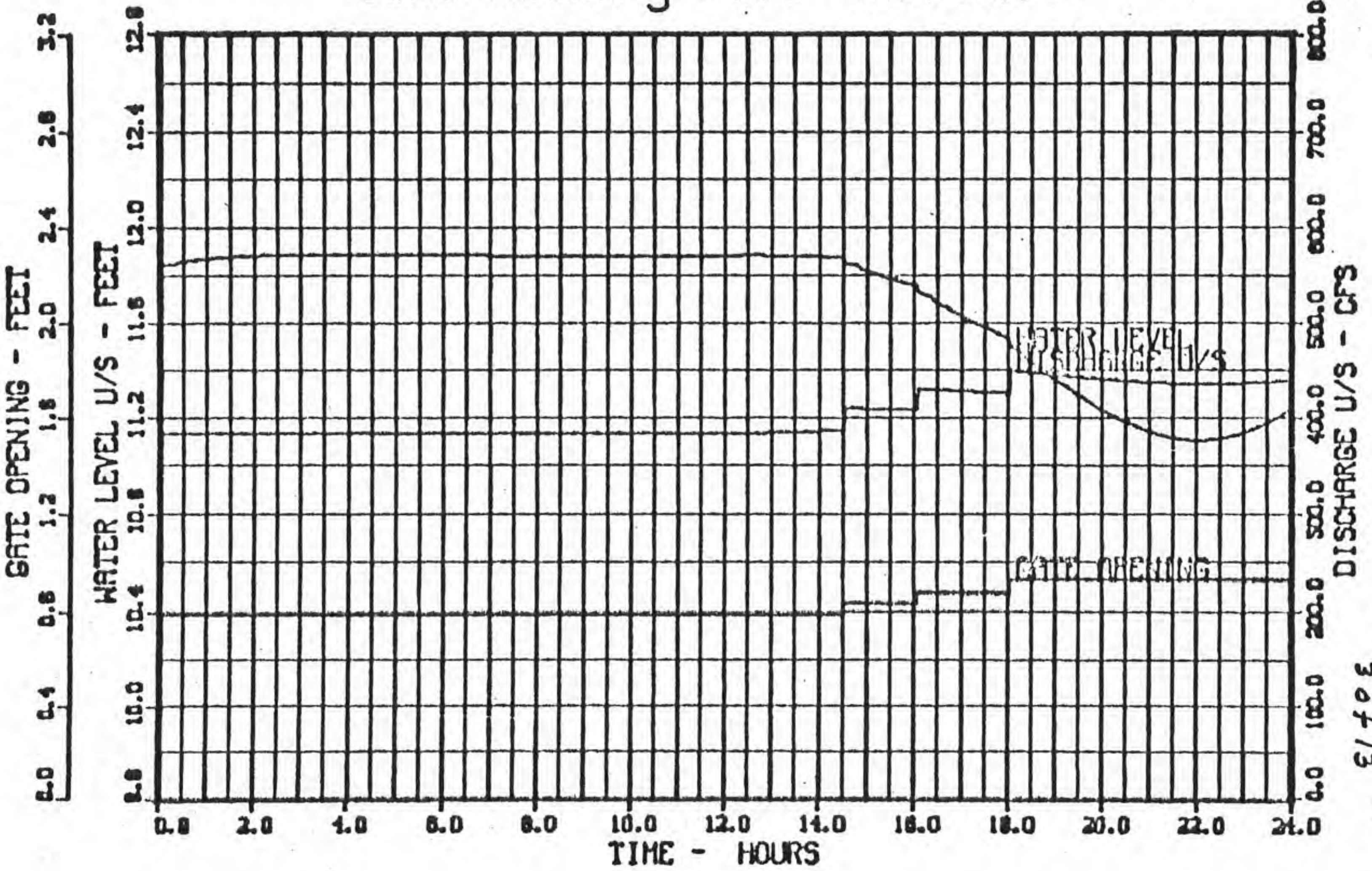
N.I.I.P. CANAL SYSTEM - CHECK NO. 3
Time History Plot Run No. 8



Automatic Downstream Control
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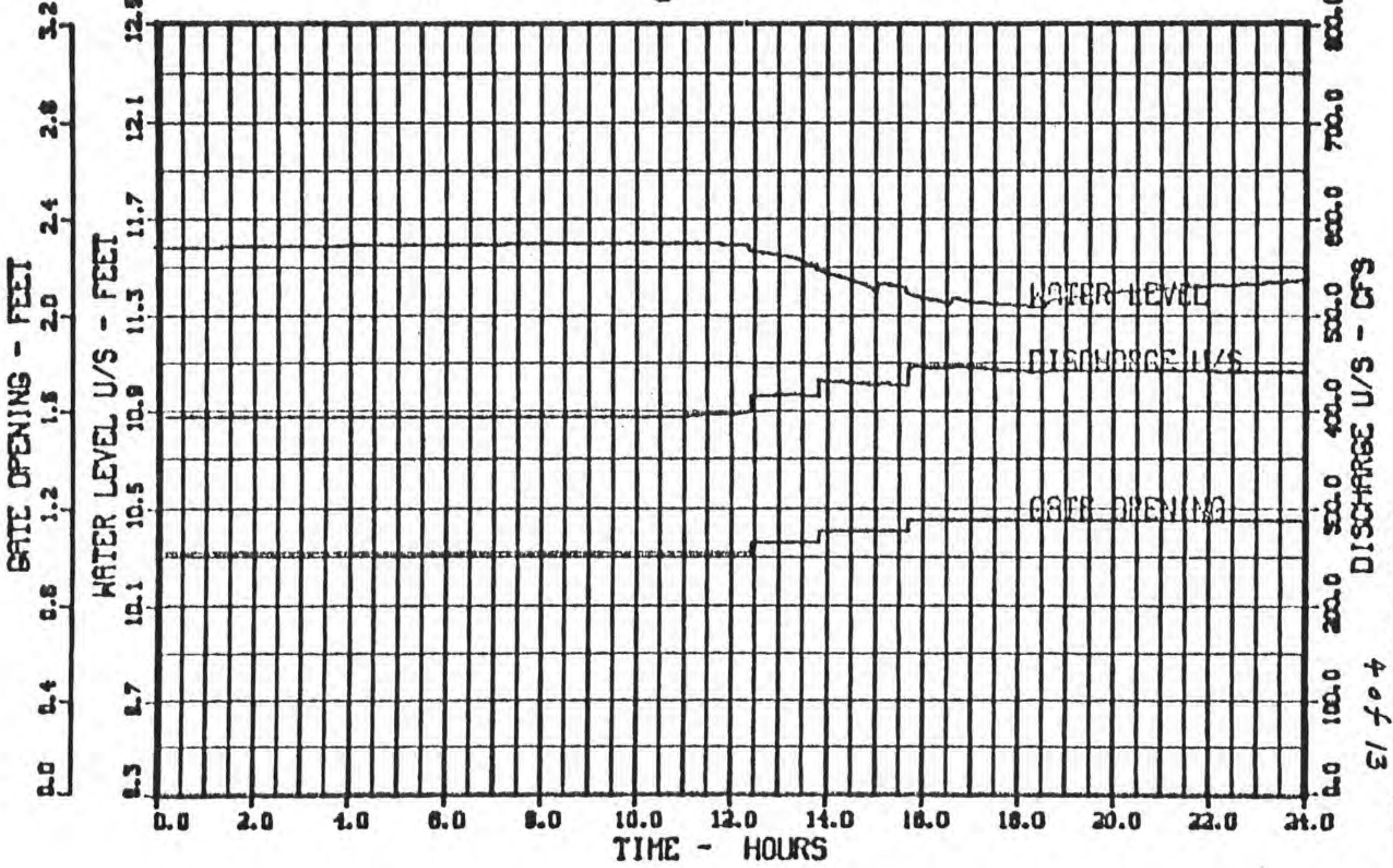
N.I.I.P. CANAL SYSTEM - CHECK NO. 4

Time History Plot Run No. 8



Automatic Downstream Control
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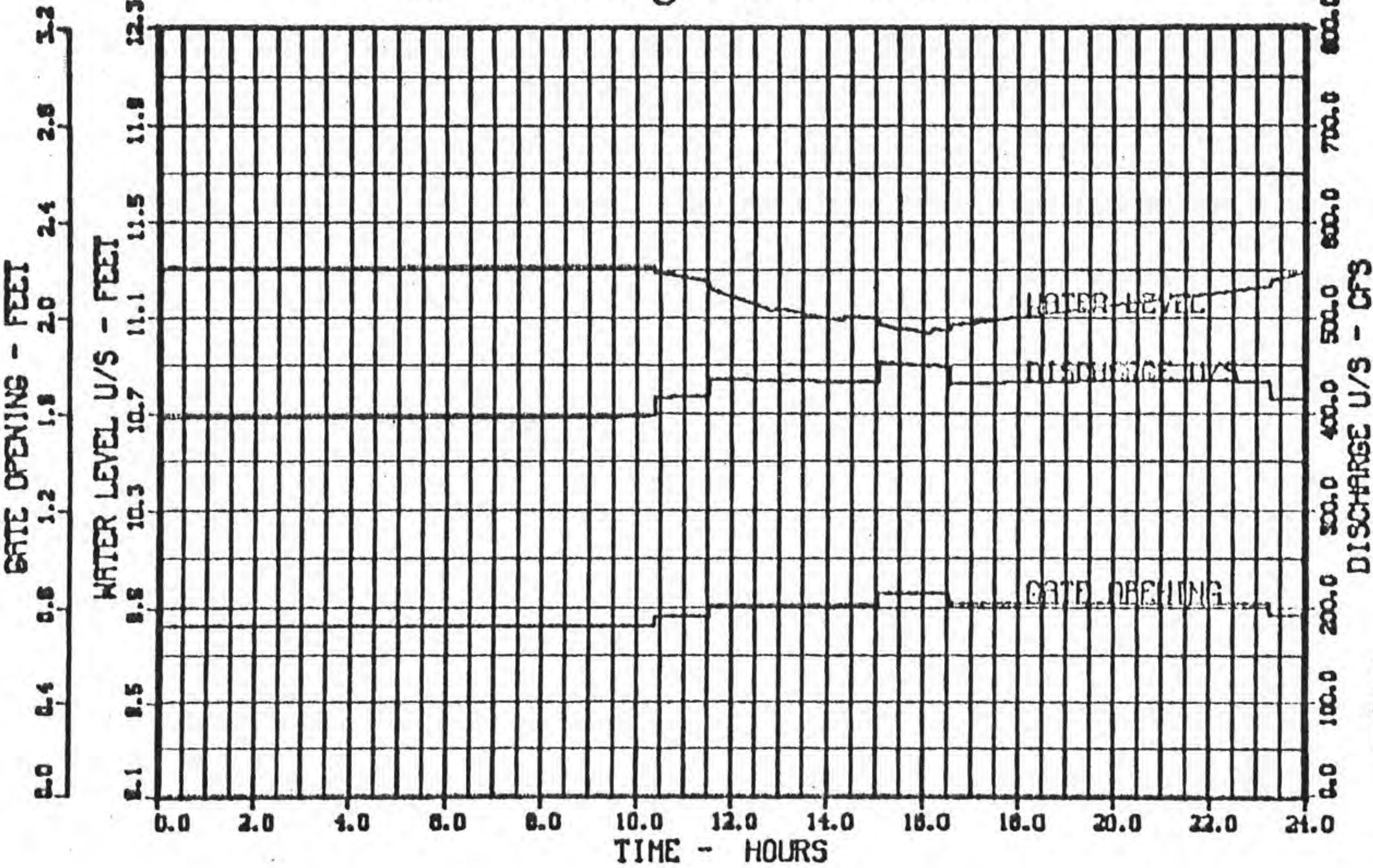
N.I.I.P. CANAL SYSTEM - CHECK NO. 5

Time History Plot Run No. 8



Automatic Downstream Control
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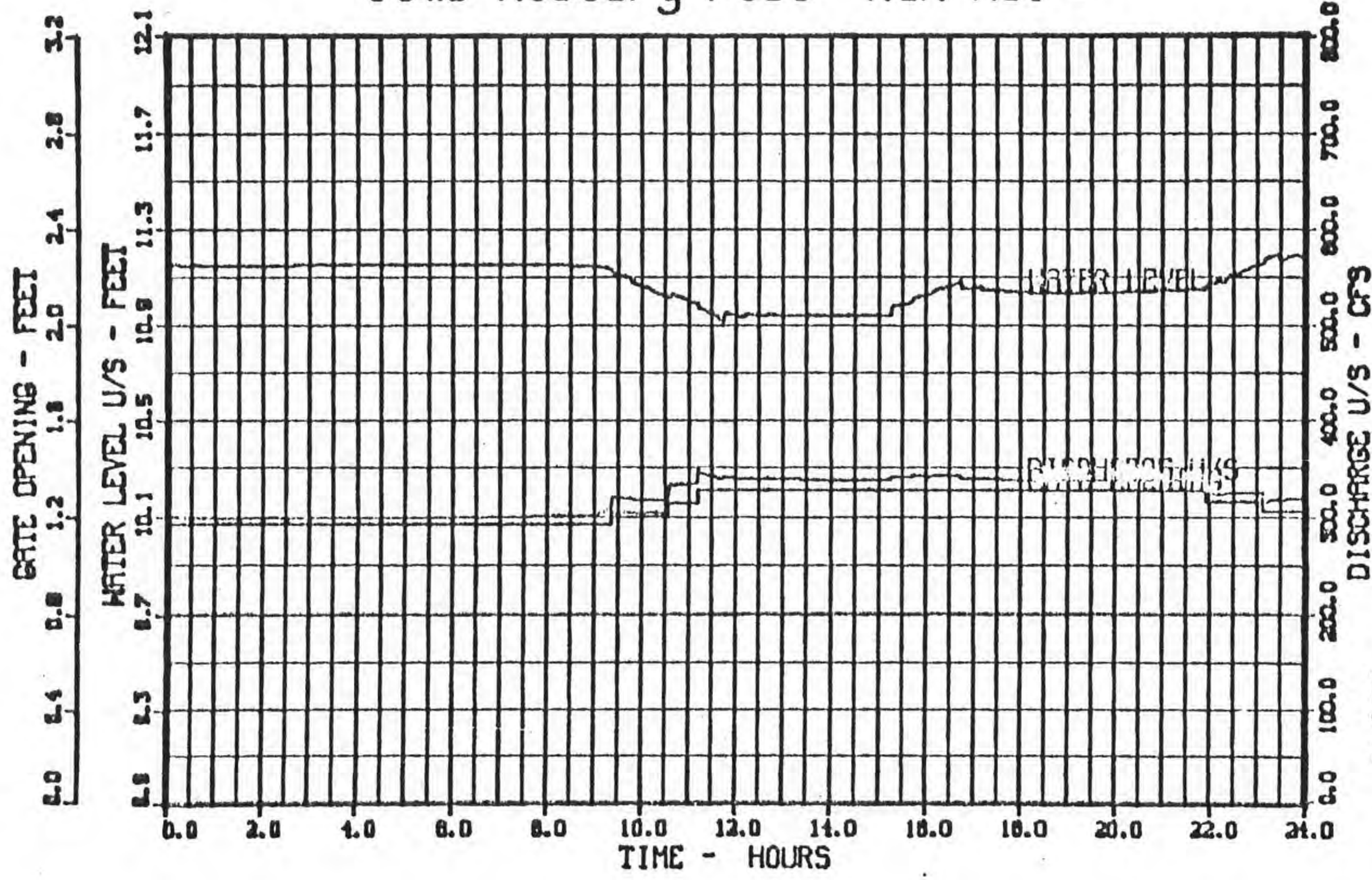
N.I.I.P. CANAL SYSTEM - CHECK NO. 6

Time History Plot Run No. 8



Automatic Downstream Control
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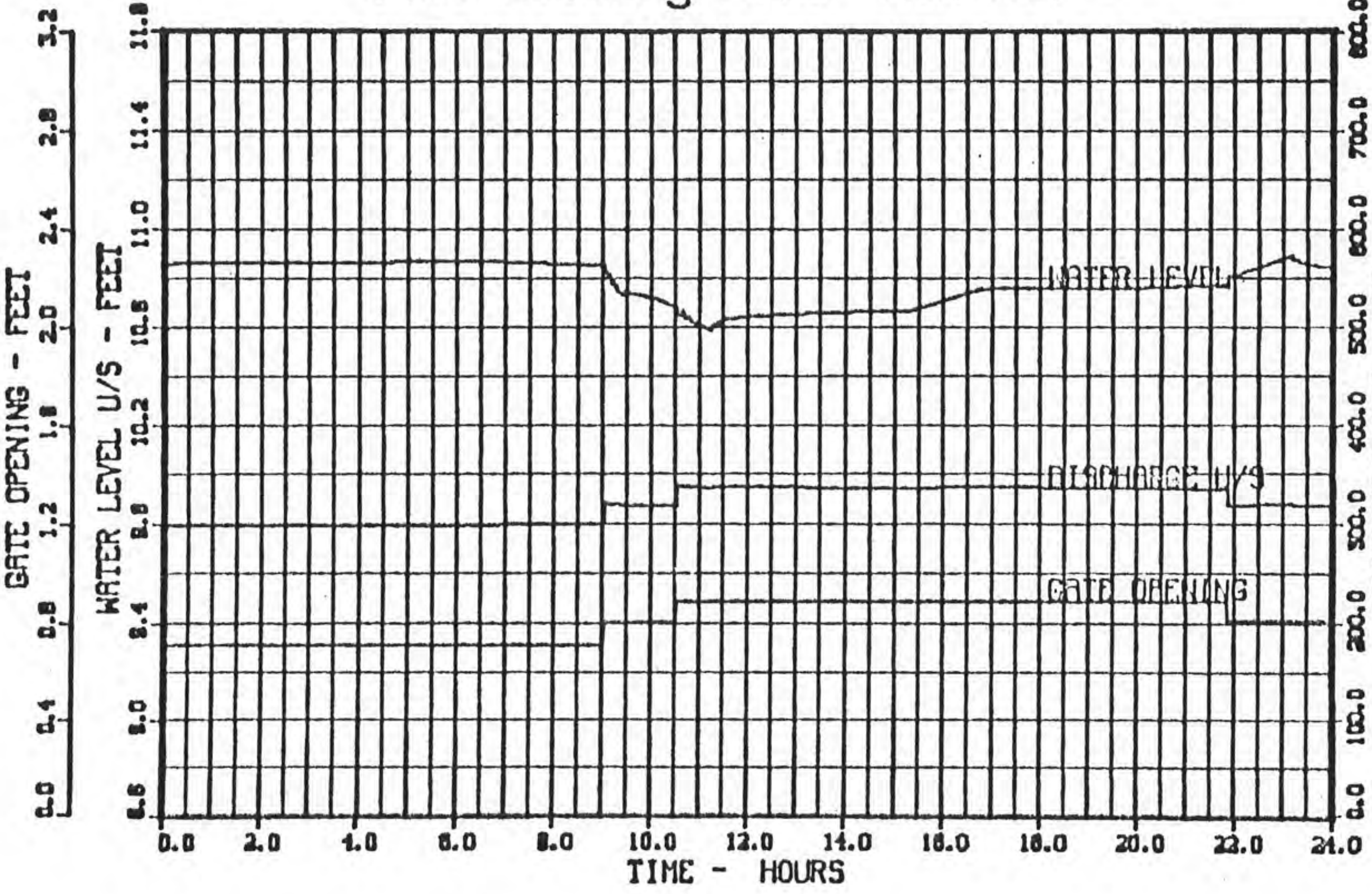
N.I.I.P. CANAL SYSTEM - CHECK NO. 7

Time History Plot Run No. 8



Automatic Downstream Control
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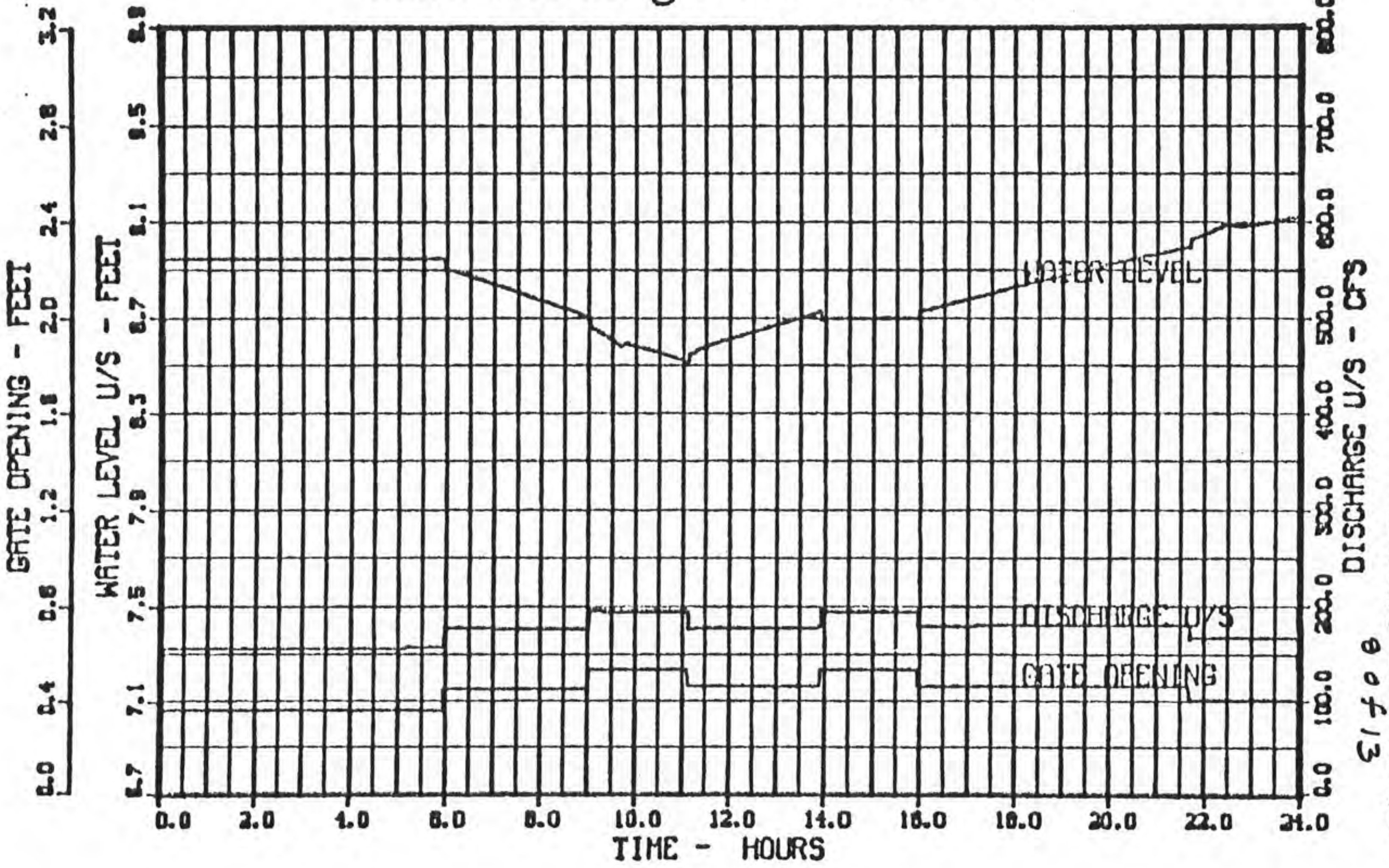
N.I.I.P. CANAL SYSTEM - CHECK NO. 8

Time History Plot Run No. 8



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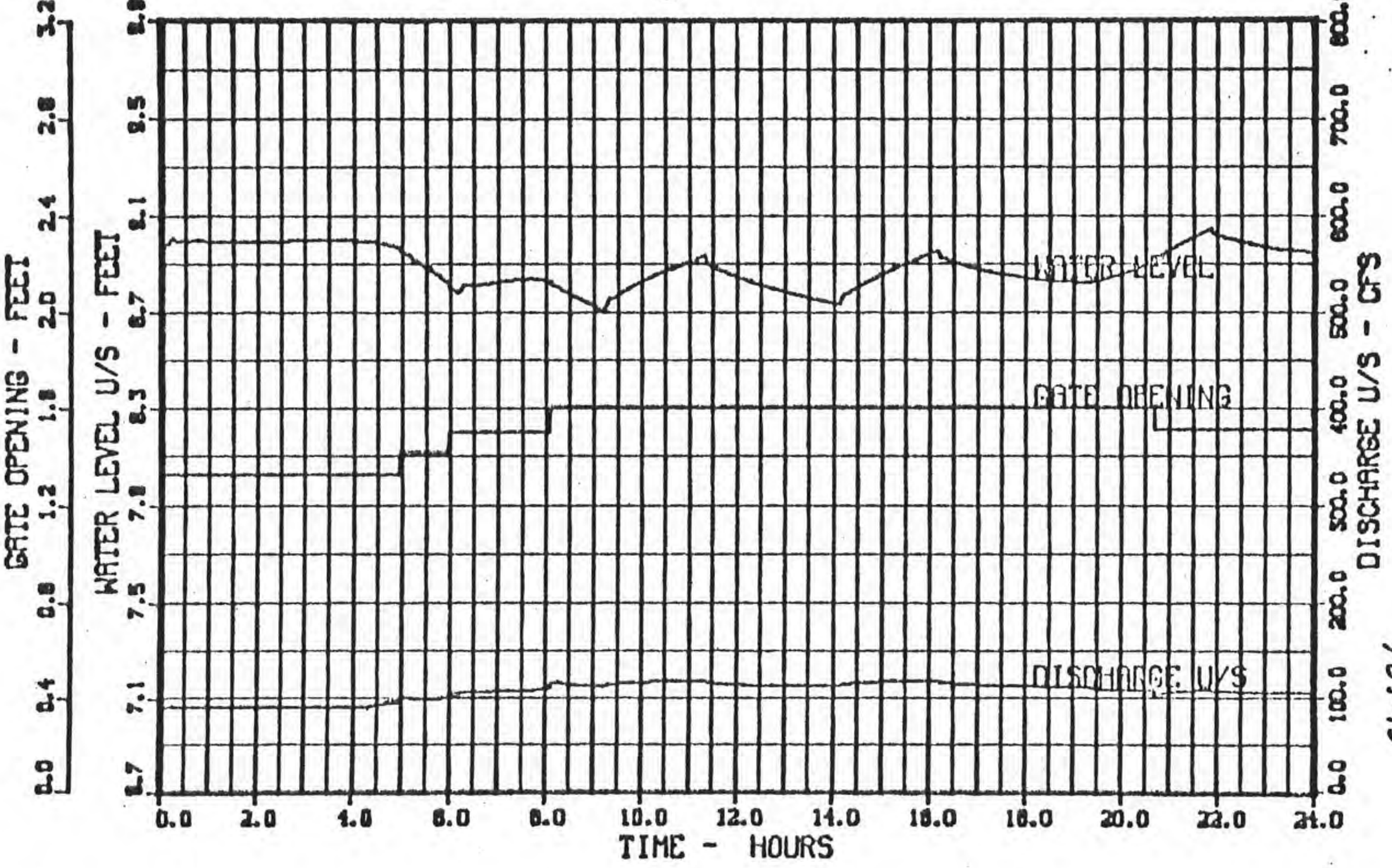
N.I.I.P. CANAL SYSTEM - CHECK NO. 9

Time History Plot Run No. 8



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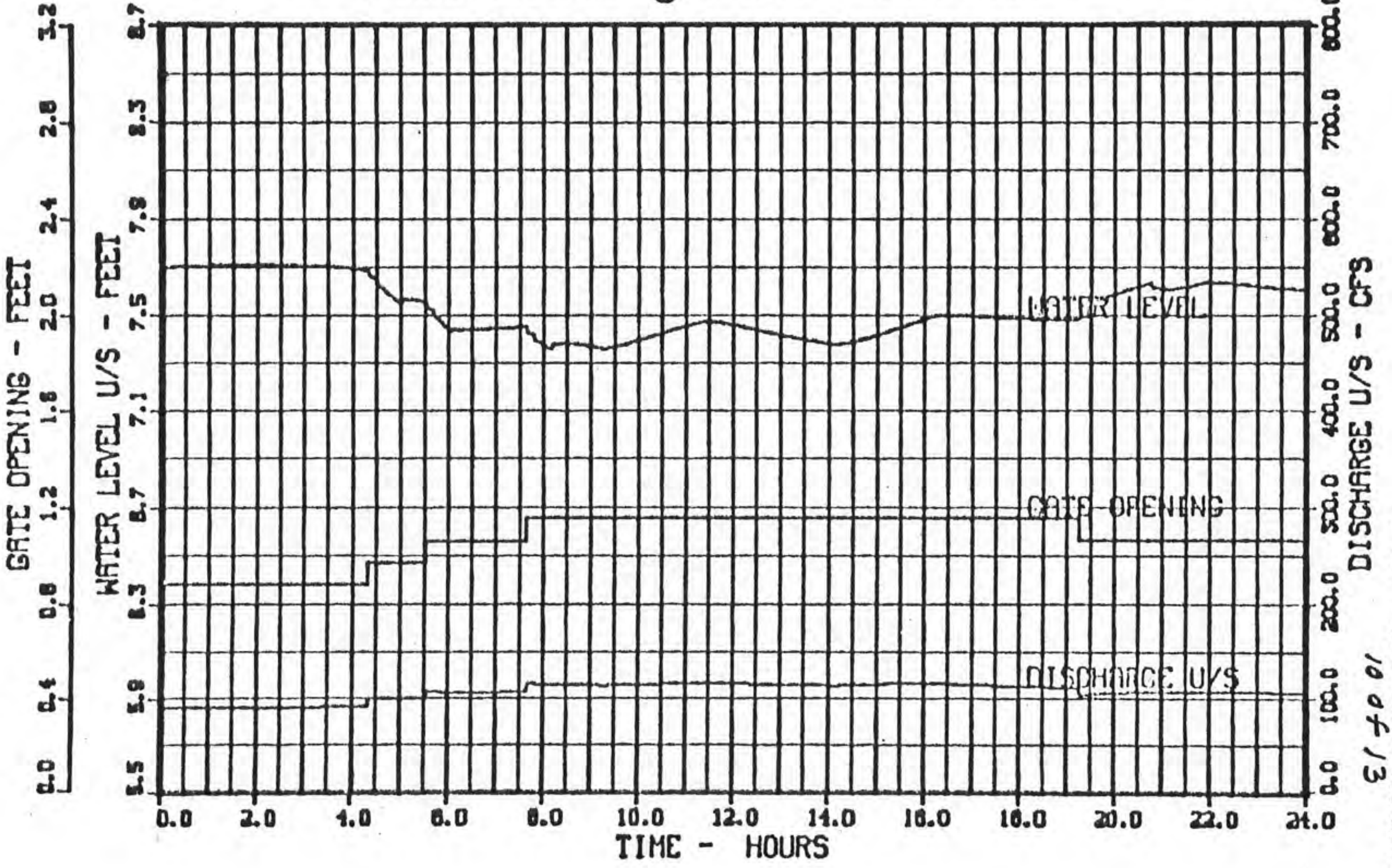
N.I.I.P. CANAL SYSTEM - CHECK NO. 10

Time History Plot Run No. 8



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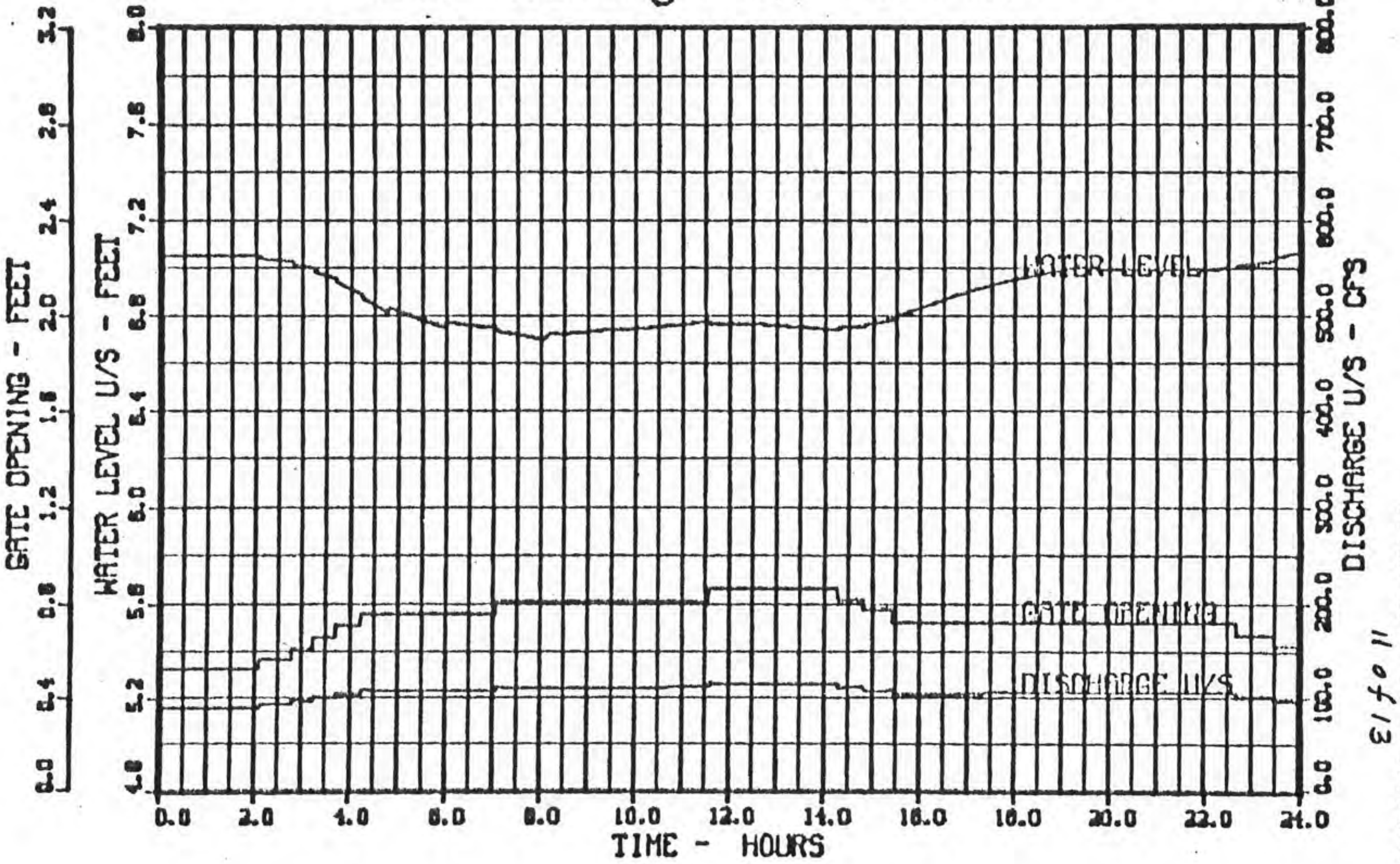
N.I.I.P. CANAL SYSTEM - CHECK NO. 11

Time History Plot Run No. 8



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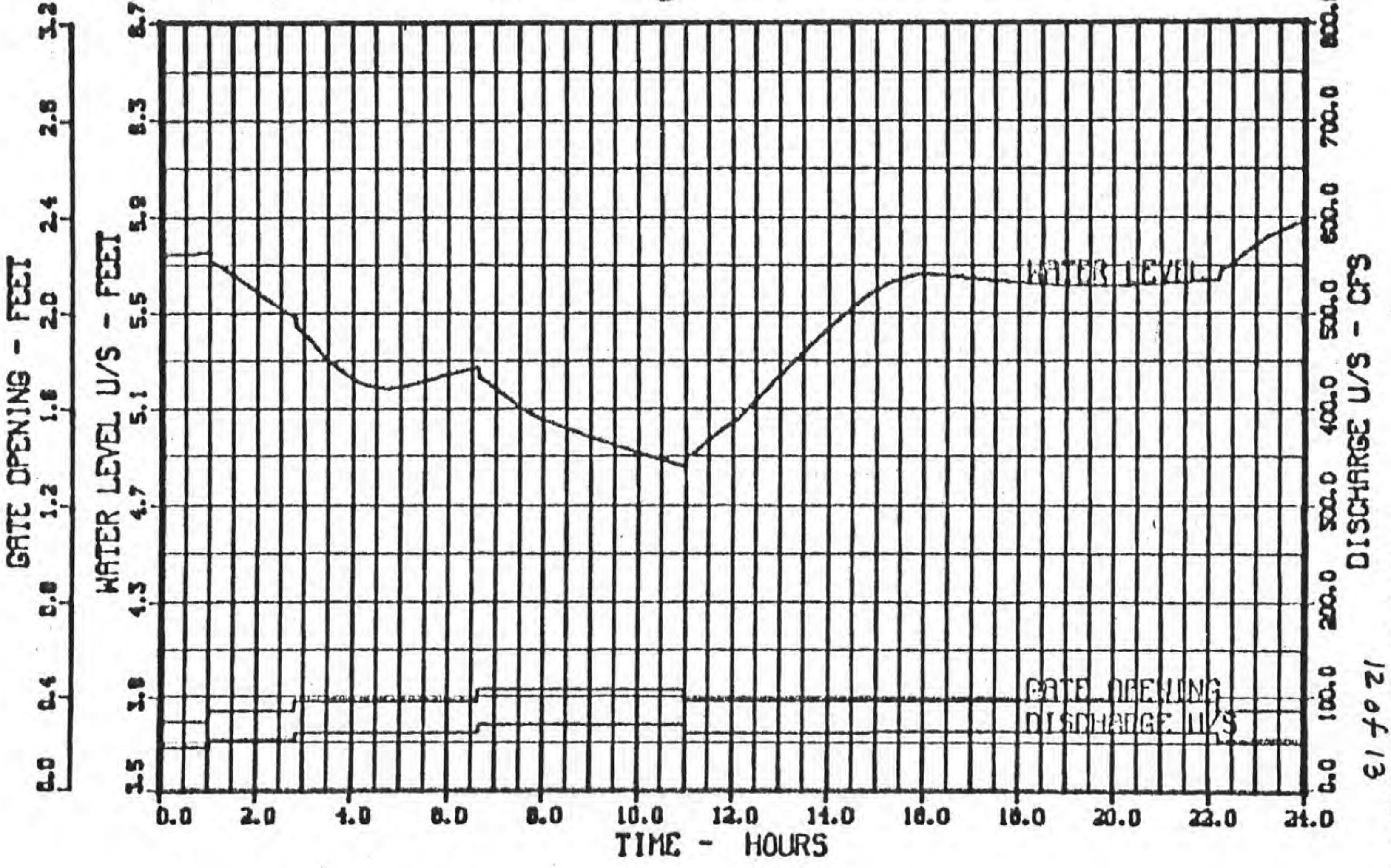
N.I.I.P. CANAL SYSTEM - CHECK NO. 12

Time History Plot Run No. 8



Automatic Downstream Control

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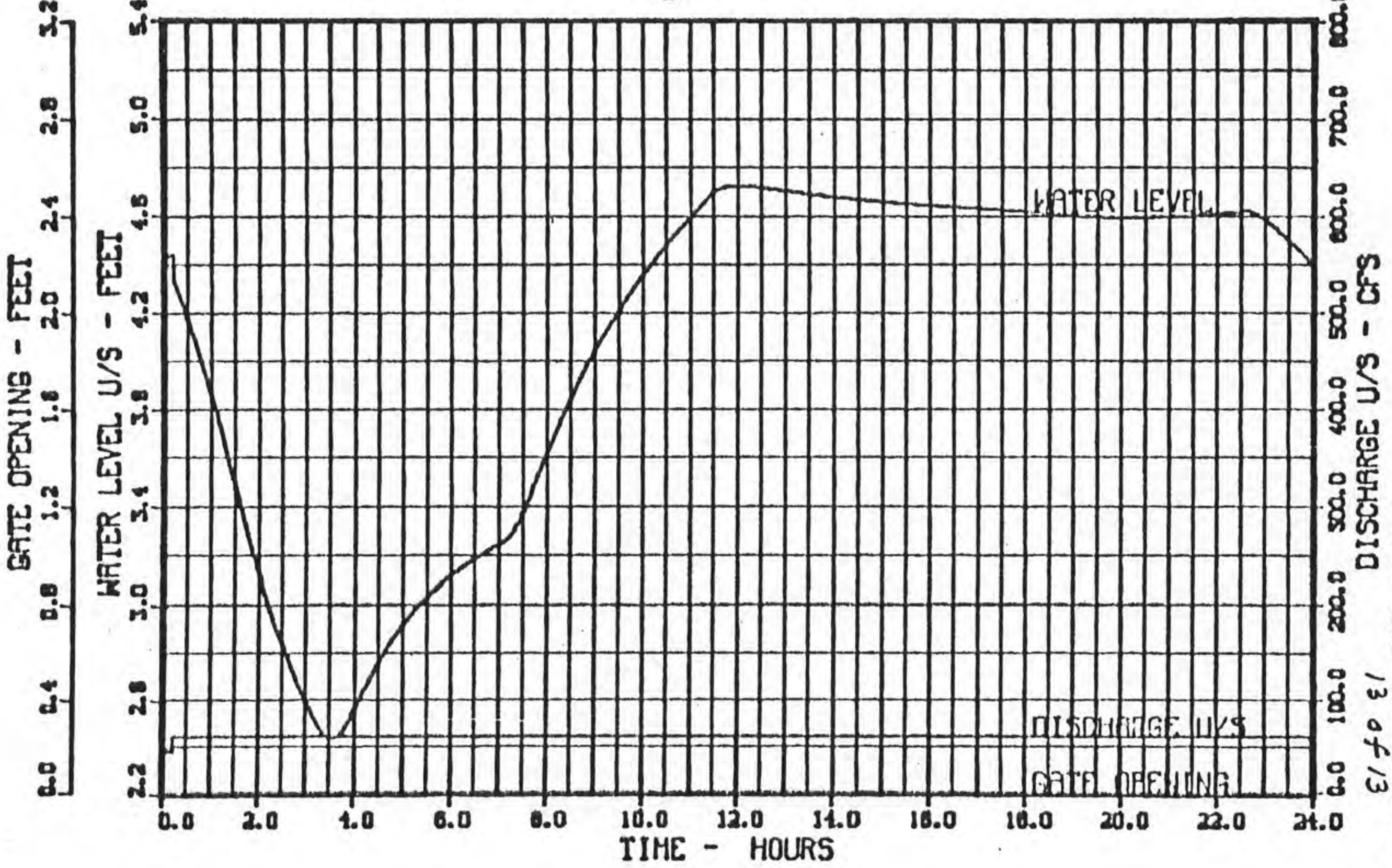
N.I.I.P. CANAL SYSTEM - CHECK NO. 13

Time History Plot Run No. 8



Automatic Downstream Control

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