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MEMORANDUM

To: Technology Development Program Manager, Dam Safety Office
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From: Shohreh Hamedian, Civil Engineer
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Subject: Dam Safety Technology Development Report DSO-2018-02 - Finite Element Analysis for Spillway Retaining Wall Shake Table Testing Program

A report on Finite Element Analysis for Spillway Retaining Wall Shake Table Testing Program, DSO-2018-02 from the Dam Safety Technology Development Program has been prepared by the Technical Service Center at the request of the Dam Safety Office. The report will be available in Adobe Acrobat Format on the Dam Safety website and will also be loaded into DSDAMS.

If you have any questions, please contact me at 303-445-3224 or at shamedian@usbr.gov.

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Finite Element Analysis for Spillway Retaining Wall Shake Table Testing Program

DSO-2018-02

Dam Safety Technology Development Program

U.S. Department of the Interior
Bureau of Reclamation
Technical Service Center
Denver, CO

January 2018

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This report presents the Finite Element (FE) analysis conducted by the Waterways and Concrete Dams Group 1. The FE analysis was performed for Shake Test Model Configuration 1 (March 30th, 2016) conducted at UCSD. Displacement and moment results from the FE analysis were compared to the measured displacement and moment output from the shake table test.

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Dam Safety Technology Development Program

Waterways and Concrete Dams Group 1, 86-68110

DSO-2018-02

Finite Element Analysis for Spillway Retaining Wall Shake Table Testing Program

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Acronyms and Abbreviations

ft	feet
g	acceleration of gravity
lb	Pounds
lb/ft^3	pounds per cubic foot
m	meter
Hz	hertz
UCSD	University of California, San Diego

Executive Summary

This report presents the Finite Element (FE) analysis conducted by the Waterways and Concrete Dams Group 1. The FE analysis was performed for Shake Test Model Configuration 1 (March 30th, 2016) conducted at UCSD. Displacement and moment results from the FE analysis were compared to the measured displacement and moment output from the shake table test.

Introduction

The UCSD shake table team conducted a spillway retaining wall shake table test during the spring of 2016 at the UCSD Englekirk Structural Engineering Center. The shake table tests were conducted to study soil structure interaction and was funded by the Dam Safety Technology Development Program. The UCSD team collected and processed the field data output from the test and also conducted an independent numerical analysis of the test in an attempt to analytically reproduce measured results.

The Waterways and Concrete Dams Group 1 developed an independent FE model of the retaining wall shake table test and compared the results of the FE model with measured field data output of the shake table test.

The purpose of this report is to document the analysis and results of the FE model developed by the Waterways and Concrete Dams Group 1. The details in this report serve to supplement and compare results from the report “Spillway Retaining Wall Shake Table Test Program: Soil-Structure Interaction” [1] developed by UCSD as part of this project. Sections 2.1 and 2.2 present background details sufficient to put the UCSD studies into proper context for development of this report; however, complete details can be found in “Spillway Retaining Wall Shake Table Test Program: Soil-Structure Interaction.”

Background

Shake Table Test

The shake table tests were performed at the large outdoor shake table under the guidance of Professor Ahmed Elgamal, Ph.D. and Kyungtae Kim, Ph.D. at UCSD. A laminar soil container was used to accommodate the spillway ground system with three different soil configurations.

Shake Table Container and Spillway Details

Figure 1 shows the laminar container on the shake table. The laminar soil box consists of 31 steel laminar frames, each separated by a steel roller system on stainless steel lined webs, to allow for uni-directional movement. Movement of the laminar frames, when subject to uni-directional dynamic loading, provides a mechanism by which energy propagating through the soil can be absorbed. This energy absorption simulates in-situ soil conditions, in which energy can propagate through a uniform soil deposit over great distances with minimal energy reflection [2]. Figure 2 shows the spillway structure model. Figure 3 provides the cross section dimensions of the spillway model with a 9.3 ft. length (out of plane dimension) and a thickness of 0.75 inches. To obtain high flexural and axial rigidities against lateral loads during tests, 12 HSS columns were welded to the base plate at a center to center spacing of 1 ft.

Figure 1. Photograph of the 15.2 ft. high, 22 ft. long, and 9.6 ft. wide laminar soil container on the shake table.

Figure 2. Photograph of the model spillway structure (9.3 ft. in length out of plane).

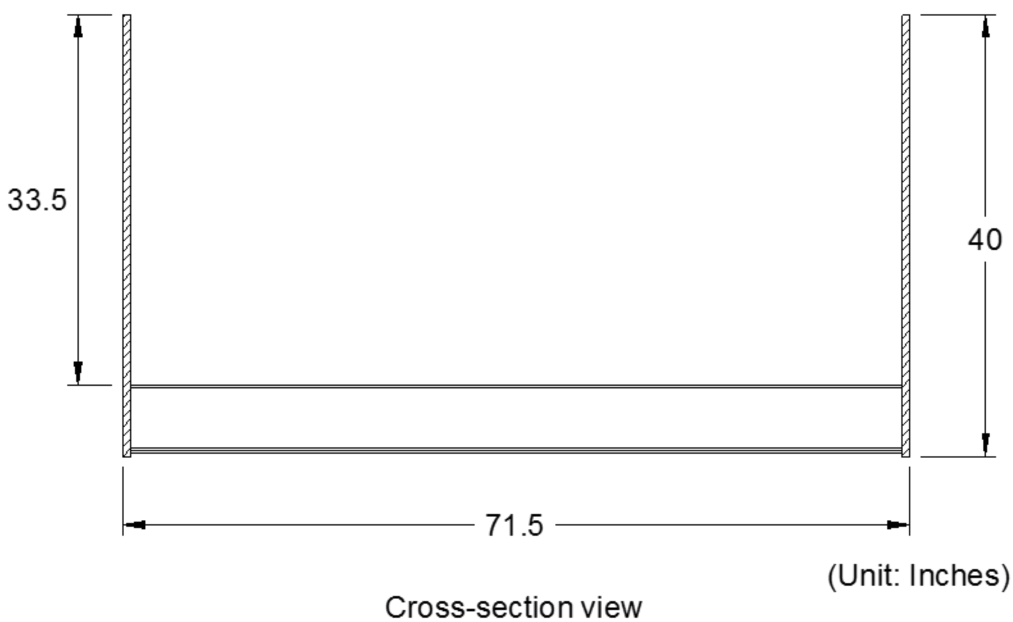

Figure 3. Dimension of the model spillway structure (9.3ft in length out of plane).

Test Model Configuration

Three different tests were performed with dissimilar backfill conditions and moisture content (W_c=7% and almost dry case). Table 1 summarizes the three test configurations employed for the shake table tests. Test Model Configuration 1 will be used for comparison with finite element results. Figure 4 shows this configuration graphically, while Table 2 presents the geotechnical properties of the soils.

Table 1. Description of test model configurations.

Test Model Configuration	Test Date	Soil configuration	
		West side	**East side**
1	3/30/2016	Very dense soil (99%†)	Dense soil (85%†)
2	4/1/2016	Very dense soil (99%†)	Clean sand
3	4/4/2016	Very dense soil (99%†)	Clean sand
† Relative compaction measured from sand cone tests Unit weight of soil: 120 pcf (very dense soil), 104 pcf (dense soil), 90 pcf (clean sand)			

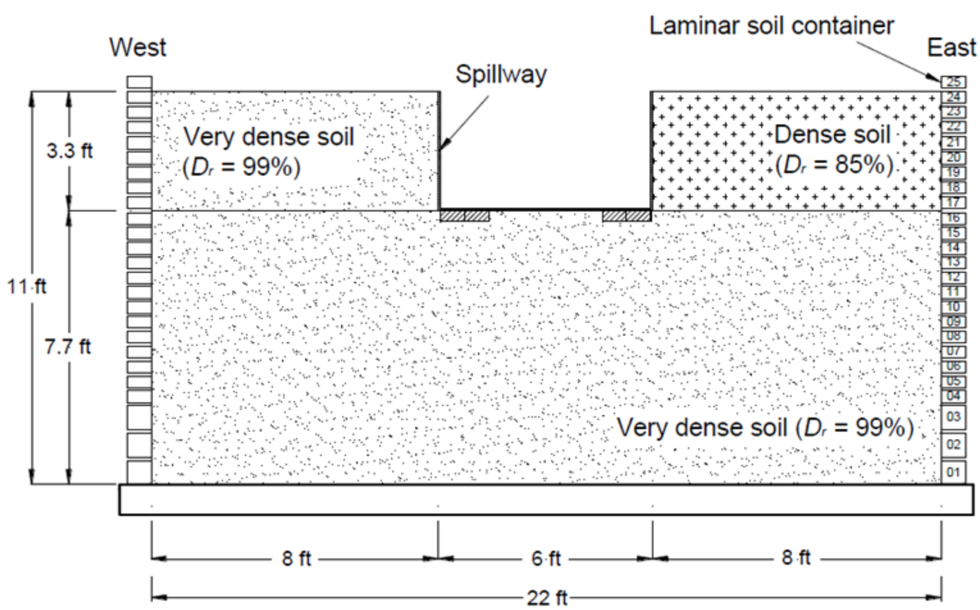

Figure 4. Test configuration (very dense soil-dense soil) performed on the 1st day (3/30/2016).

Table 2. Soil Material Properties

Layer	Depth (ft)	Thickness (ft)	Soil 1 (stiff soil)			Soil 2 (soft soil)		
			V_s (ft/s)	G (psi)	S_u (psi)	V_s (ft/s)	G (psi)	S_u (psi)
1	0 – 1.7	1.7	313	2551	0.35	238	2551	0.2
2	1.7 – 3.3	1.7	313	2551	1.03	238	2551	0.4
3	3.3 - 4.8	1.5	454	5392	1.67			
4	4.8 - 6.3	1.5	410	4384	2.29			
5	6.3 - 7.8	1.5	410	4384	2.91			
6	7.8 - 9.3	1.5	491	6286	3.53			
7	9.3 - 11	1.7	491	6286	4.19			

Note: 1. Unit weight of soil 1 and 2 = 120 pcf and 105 pcf, respectively
2. Poisson's ratio of soil 1 and 2 = 0.3
3. Friction angle of soil 1 and 2 = 53.2 and 40 degrees. Respectively
4. For shear strength, $S_u = \sigma_m \sin(\phi)$ where confinement, $\sigma_m = (\sigma_v + 2\sigma_h)/3$ where $\sigma_v = \gamma H$, H = depth, $\sigma_v = K_o\, \sigma_v$, $K_o = \nu/(1\text{-}\nu)$, ν = Poisson's ratio

Earthquake Input motions

The recorded Northridge earthquake motion and the Takatori earthquake motion were used as input excitation for the shake table tests. To further investigate the seismic response under different ground accelerations, the original Northridge and Takatori earthquake motions were scaled in terms of: i) time duration by factors of 1.0/2.5 and 1.0/5.2, and ii) amplitude by a factor of 2 for the Northridge earthquake motion. Table 3 summarizes the measured peak acceleration, velocity, and displacement at the test model base and highlights Nor100PT0 as the event chosen for comparison in the Reclamation finite element model analysis.

Table 3. Earthquake Input Motions

No.	Input motion	Earthquake	Amplitude scale factor	Time scale factor	GPA† (g)	PGV† (in/s)	PGD† (in)
1	Nor100PT0	Northridge	1	1	0.50	11.39	2.61
2	Nor100PT1		1	1.0/2.5	0.40	4.28	1.05
3	Nor100PT2		1	1	5.2	0.39	2.09
4	Nor200PT1		2	1.0/2.5	0.94	8.51	0.66
5	Tak100PT0	Takatori	1	1	0.67	52.63	13.69
6	Tak100PT1		1	1.0/2.5	0.67	20.63	2.35
7	Tak100PT2		1	1.0/5.2	0.58	8.84	0.49

†Measured at the test model base during the 1st test (stiff-soft soil configuration)

Shake Table Results

The Northridge earthquake scenario (Nor100PT0) performed on Day1 will be used to compare results. Specifically, relative displacements at the top of the walls to the displacement along the bottom of the walls along with moments at the bottom of the walls will be evaluated.

UCSD Finite Element Analysis

FE model configuration

System modeling was performed by UCSD using a 2D plane strain configuration (thickness of 1 inch). The OpenSees platform, an object oriented, open-source FE analysis frame work was used. Figure 5 shows the 2D FE mesh of the shake table test model.

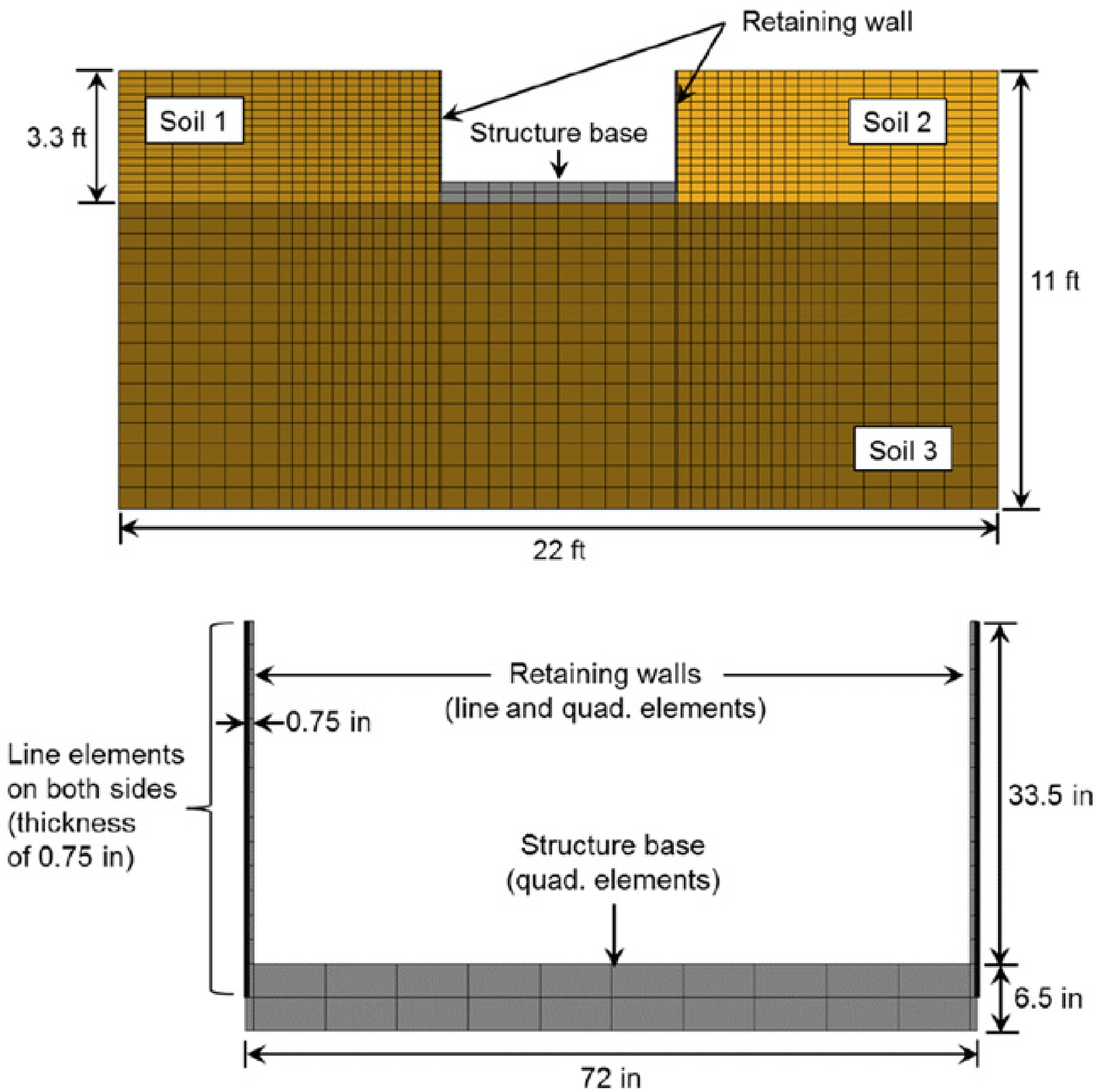

Figure 5. FE mesh for shake table test model: (a) entire structure-ground model and (b) structure model

Soil element material properties

Four node quadrilateral elements were used to model the backfill soil (Figure 5b). A total of 32 layers were specified along the soil depth. Each layer was modeled as a nonlinear hysteretic material with a Von Mises multi-surface kinematic plasticity model (PressureIndependMutliYield), as shown in Figure 6. This material models the soil hysteretic elasto-plastic shear response and the accumulation of any permanent deformation. The nonlinear shear stress-strain backbone curve (Figure 6b) is represented by the hyperbolic relationship defined by two material constants: low strain shear modulus and ultimate shear strength; (1) the shear modulus was defined using a parabolic relationship and (2) the peak shear strength increased with depth based on the friction angle. The following equation was used to define G_{max} of each layer based on the confining stress (σ_m) at the middle depth of each layer

$$Gmax = 1000\ K\ \sigma_m^{\ 0.5}\ (lb/ft^2)$$

where the value of K was determined to be the average of V_s=square root of G_{max} divided by soil density, as shown in Table 4.

For soil layers above the level of the wall base, a constant value of S_u was defined as shown in Figure 7. Based on the triaxial test, this constant value was chosen to be average of S_u in the possible range of confining stress for the FE model. For soil layers below the structure base, S_u linearly varied with increasing confining stress along depth according to the relationship $S_u = c + \sigma_m \sin(\varphi)$, where c is cohesion, σ_m is confining stress along soil layer, φ is friction angle as shown in Figure 7. This appears to be a departure from the original soil properties as shown previously in Table 2.

Table 4. Average shear wave velocity evaluated from the recorded acceleration for Model 1

Part	Location	Model 1		
		Backfill material	Shear wave velocity	Value of K^*
Soil 1	West of wall	Very dense soil	198 ft/s	14.0
Soil 2	East of wall	Dense soil	120 ft/s	5.85
Soil 3	Below wall	Very dense soil	360 ft/s	22.4

*value of K for $G_{max} = 1000\ K\ \sigma_m^{\ 0.5}$ in psf was determined to have the average shear wave velocity where σ_m is the confining stress in lb/ft^2

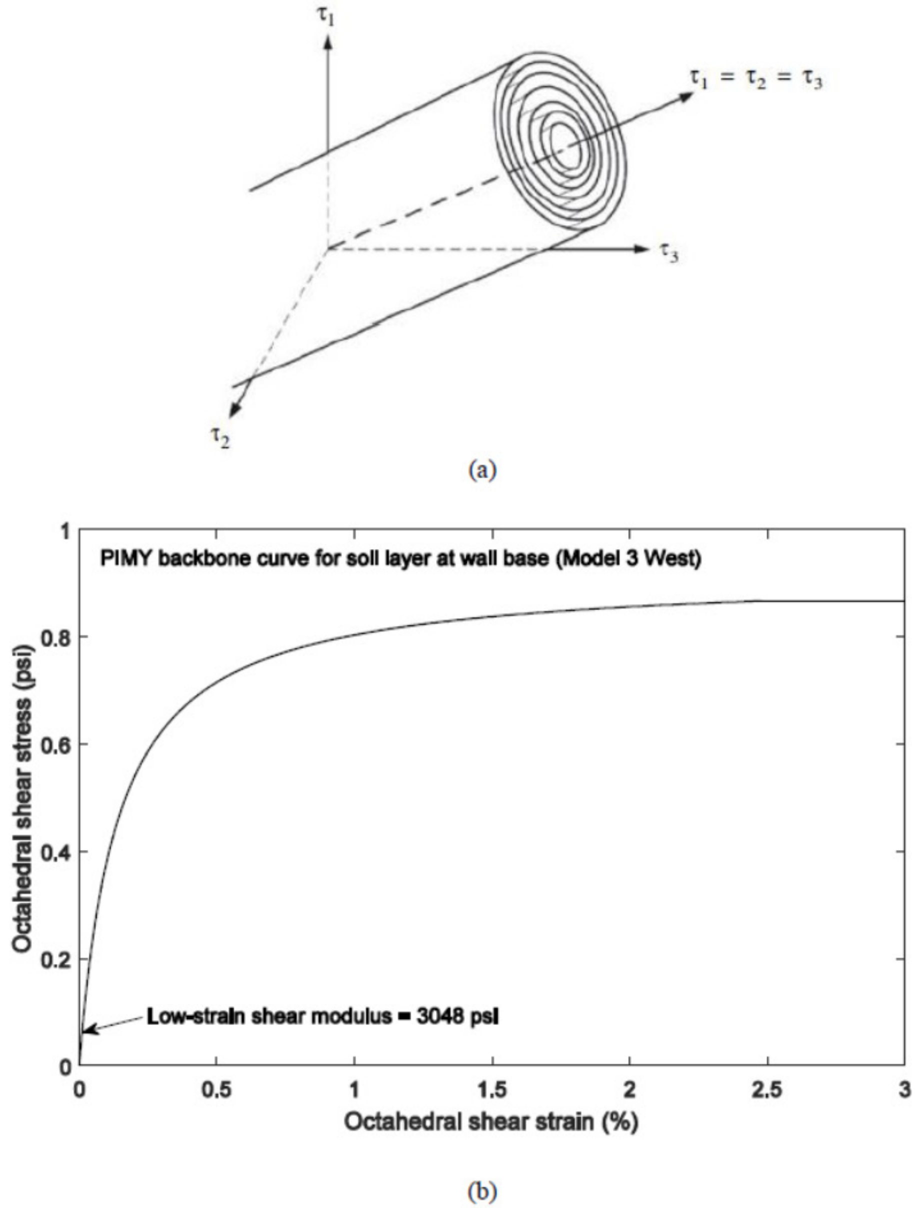

Figure 6. Pressure IndepenMulti Yield (PIMY) material properties: (a) Von Mises multi-yield surfaces of J2 plasticity model and (b) material backbone curve used for the soil layer at the wall base

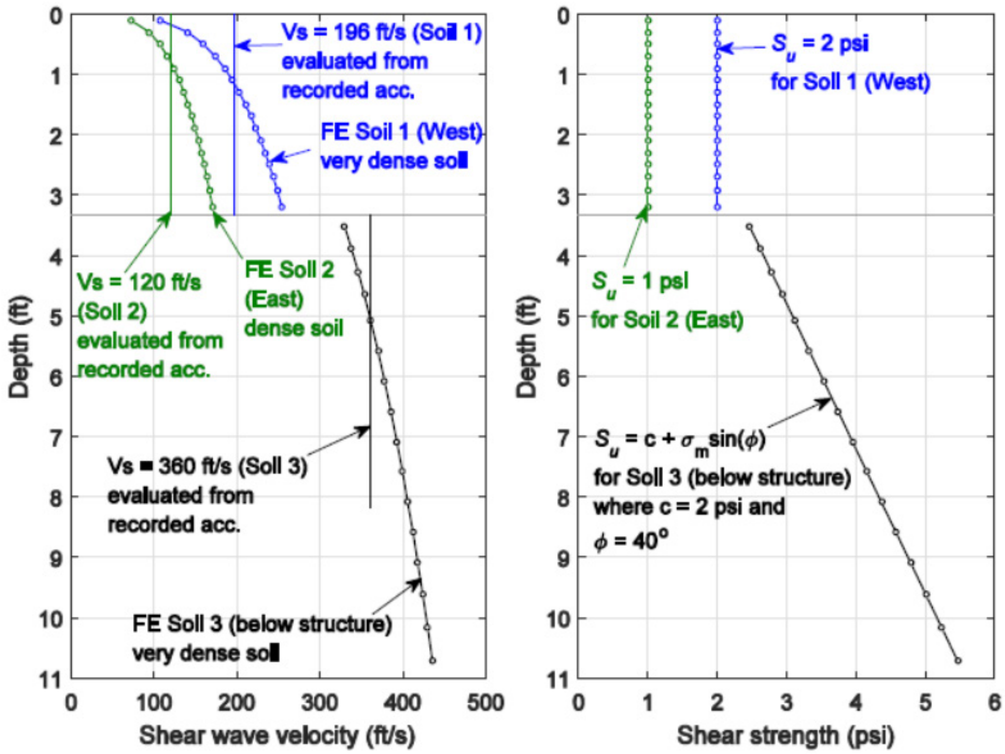

Figure 7. FE material properties of the soil layer (32 layers in total) for Model 1: variation of shear wave velocity (left) and shear strength (Su; right) along depth (white dots indicate middle depth of 32 soil layers), along with average shear wave velocities evaluated from the recorded soil acceleration for Nor100PT0

Reclamation Finite Element Model

2D Finite Element Model Description

The Reclamation finite element model modeled using LS-DYNA is detailed in this section. Figure 8 shows the instrumentation layout of accelerometer (labeled A) and string potentiometer (labeled SP), a string potentiometer is a transducer used to detect and measure linear position and velocity using a flexible cable. The recorded displacement time history from the instrumentation was used in the finite element analysis in an attempt to replicate the shake testing. In order to model the sensor locations, the soil was modeled as layers and time history displacements were applied to the edge nodes. The soil below the spillway was modeled as five separate layers. Similarly, the soil adjacent to the west and east wall were modeled as two separate layers. The layered finite element model is shown in Figure 9.

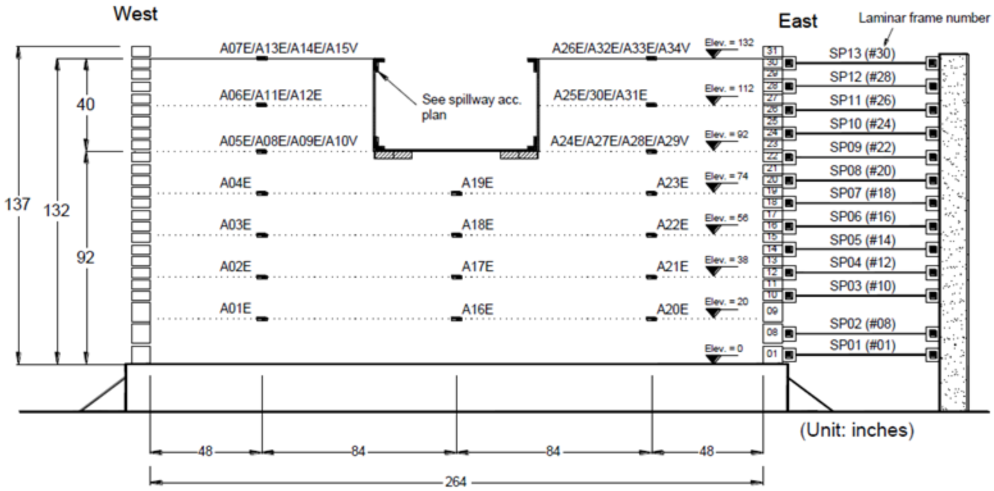

Figure 8. Instrumentation layout of accelerometer and string potentiometer

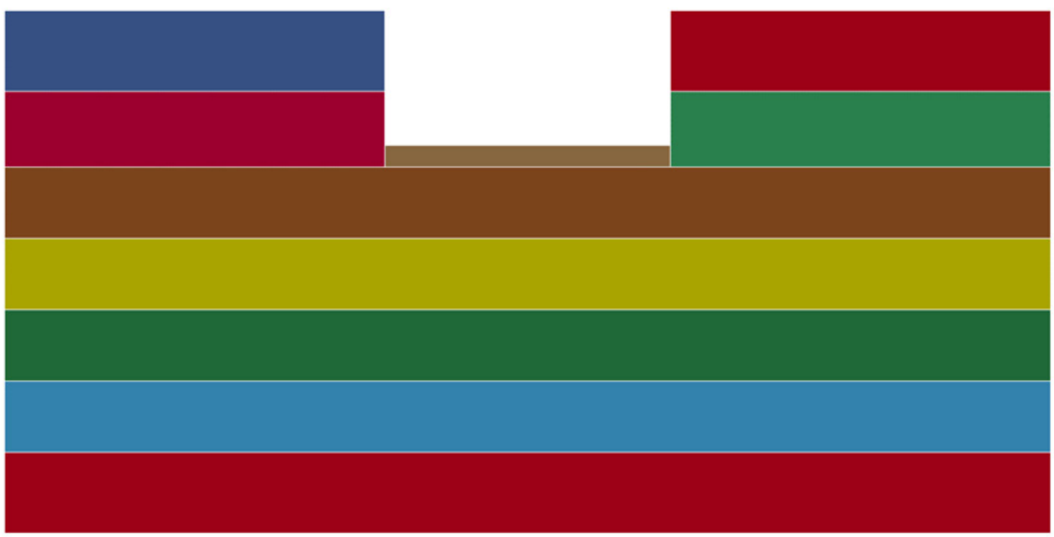

Figure 9. FE model soil material modeled as several layers to replicate the physical shake table sensors

The soil layers were modeled as solid elements. Four node quadrilateral elements were used to mesh the soil layers and is shown in Figure 10. The elements were approximately 2.66 inch x 1.8 inch in size.

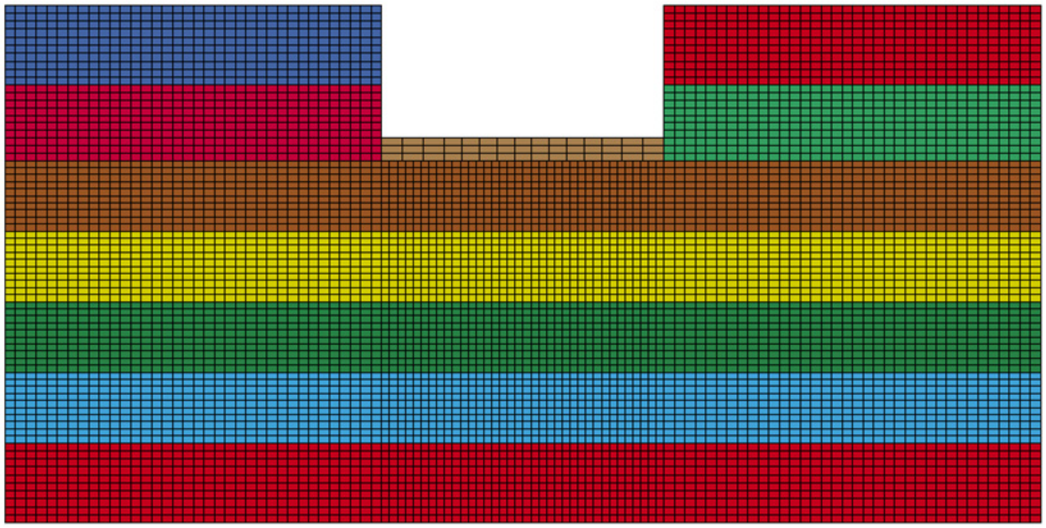

Figure 10. FE mesh for soil layers

The spillway structure finite element model is shown in Figure 11. The walls were modeled as shell elements. The left wall is shaded red and the right wall is shaded blue. The bottom was also modeled with shell elements and represents the bottom slab and stiffeners. The wall elements were approximately 2 inches x1.87 inches in size. Similarly the bottom slab elements were approximately 5.33 inches x 1.83 inches in size.

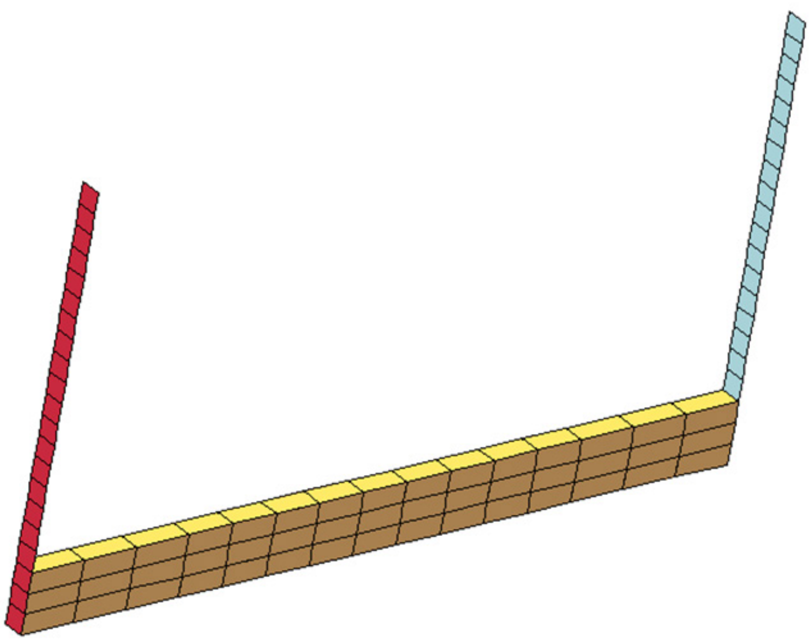

Figure 11. FE model for retaining wall

Material Properties

The soil material was modeled using material MAT_PSEUDO_TENSOR in LS DYNA. This model can be used as a simple tabular pressure-dependent yield surface. This model is suited for implementing standard geologic models like the Mohr-Coulomb yield surface with a Tresca limit as shown in Figure 12. This material combined with a tabulated compaction equation of state (described below) has been used very successfully to model ground shocks and soil structure interactions up to approximately 1.5 x 10^6 psi [3].

The basic difference between this soil material model and the one used by UCSD is that with this material model the input is basically a Mohr-Coulomb failure envelope, while UCSD input involves a shear stress/strain relationship.

The tabulated compaction model is a linear internal energy model. Pressure is defined by $p = C(\varepsilon_v) + \gamma T(\varepsilon_v)E$ in the loading phase . Unloading occurs along the unloading bulk modulus curve to the defined pressure cutoff. Reloading always follows the unloading path to the point where unloading began, and continues on the loading path as show in Figure 13 [3].

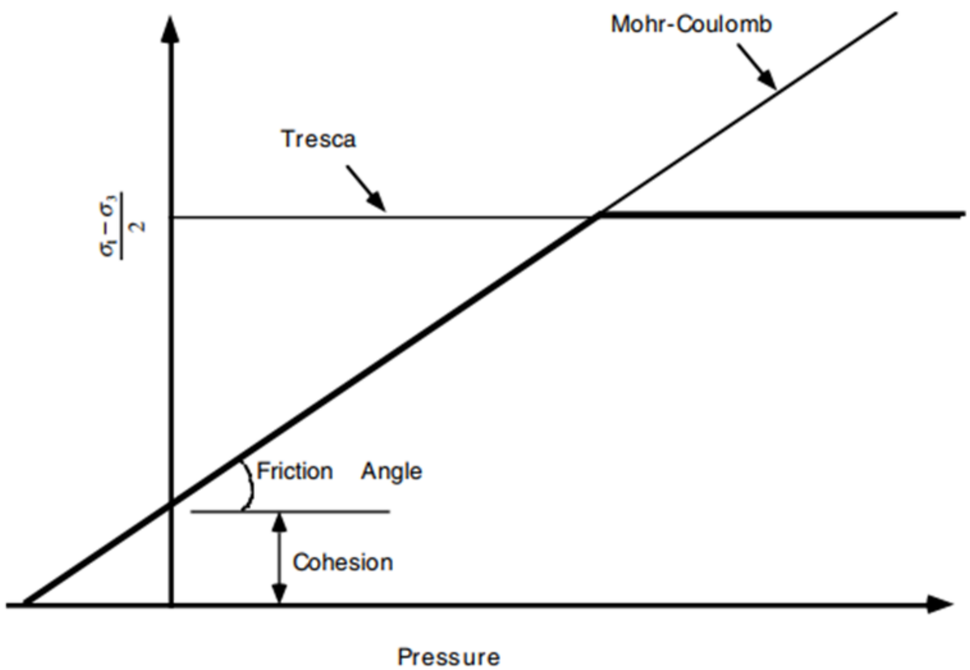

Figure 12. Mohr-Coulomb surface with Tresca limit

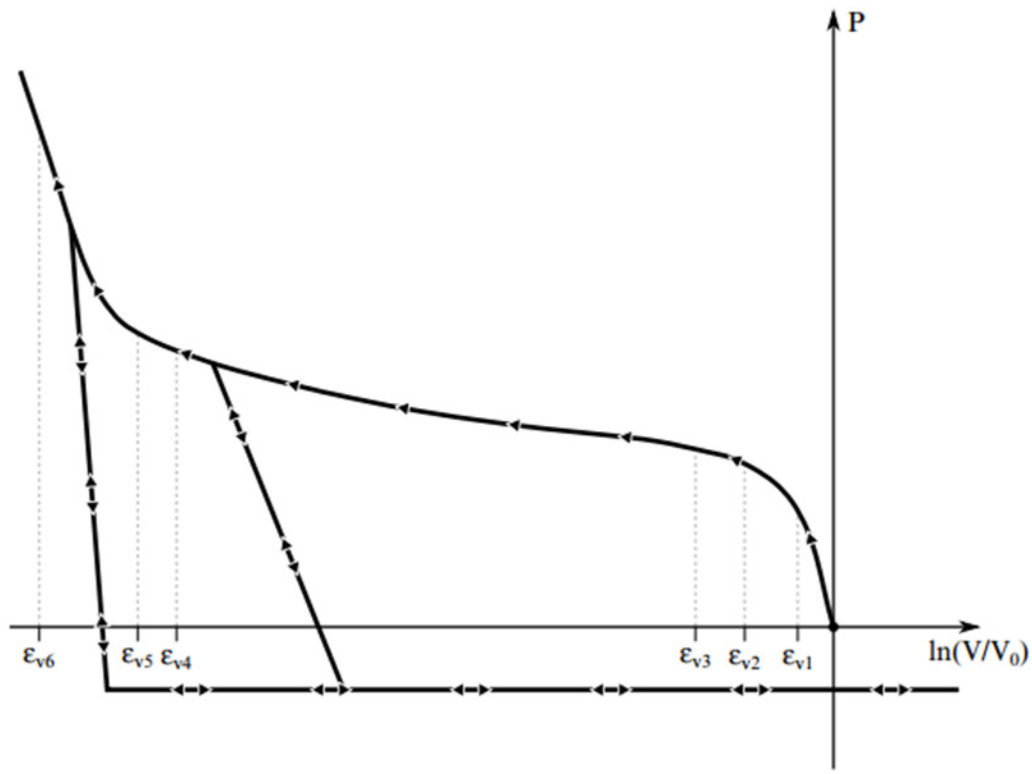

Figure 13. Pressure Versus Volumetric strain curve for Equation of State with compaction.

Elastic Material (MAT_ELASTIC) was used to model the spillway in LS-DYNA. Steel material properties were used to model the spillway:

a) Density of 490 $\frac{lb}{ft^3}$

b) Young's Modulus of 29 x 10^6 $\frac{lb}{in^2}$

c) Poisson Ratio of 0.3

Contact surface properties

The contact between the spillway and the soil was modeled with a contact surface as shown in Figure 14.

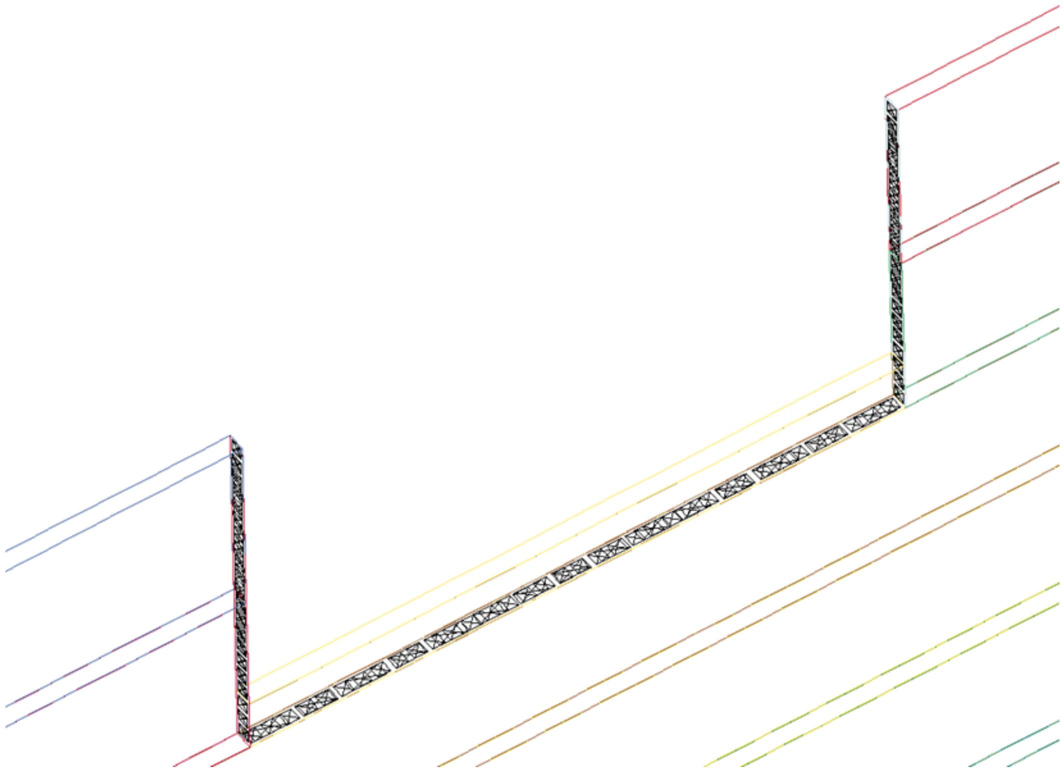

Figure 14, Contact Surface between Spillway and Soil

Damping

Damping is inherent when using a nonlinear finite element model, due to friction losses at contact surfaces and yielding of the soil. Additional damping is sometimes required in a finite element model in order to model the damping which exists in the physical domain being modeled.

LS-DYNA uses Rayleigh damping. In traditional Rayleigh damping, a global damping matrix C is constructed as a linear combination of the mass matrix M and the stiffness matrix K as:

$C = \alpha (M) + \beta (K)$

Where α is the mass proportional damping factor and β is the stiffness proportional damping factor. The mass portion of the global damping decreases with increasing frequency, while the stiffness portion of the global damping increases with increasing frequency.

Global Damping:

The proper amount of damping to apply to the model depends on its measured response to an applied excitation. A global mass-proportional Rayleigh damping value of α equal to 3 was used in this analysis. This α value equates to mass damping up to 10% for frequencies of two cycles per second and higher.

Damping Part Stiffness:

Values of 0.01 and 0.25 are recommended for proprietary Rayleigh damping coefficient for stiffness weighted damping. A value of 0.25 was selected for this analysis.

Dynamic Results

FE analysis results

Reclamation finite element results are summarized in this section. Original soil material properties from Table 2 were used. Figure 15 and Figure 16 show the displacements at the top of the west and east walls respectively, while Figure 17 and Figure 18 show the moments at the bottom of the west and east walls respectively. In each figure there are three sets of results. The blue line shows Reclamation finite element results. The red line shows data from the shake table test itself, while the green line shows data from the UCSD finite element results.

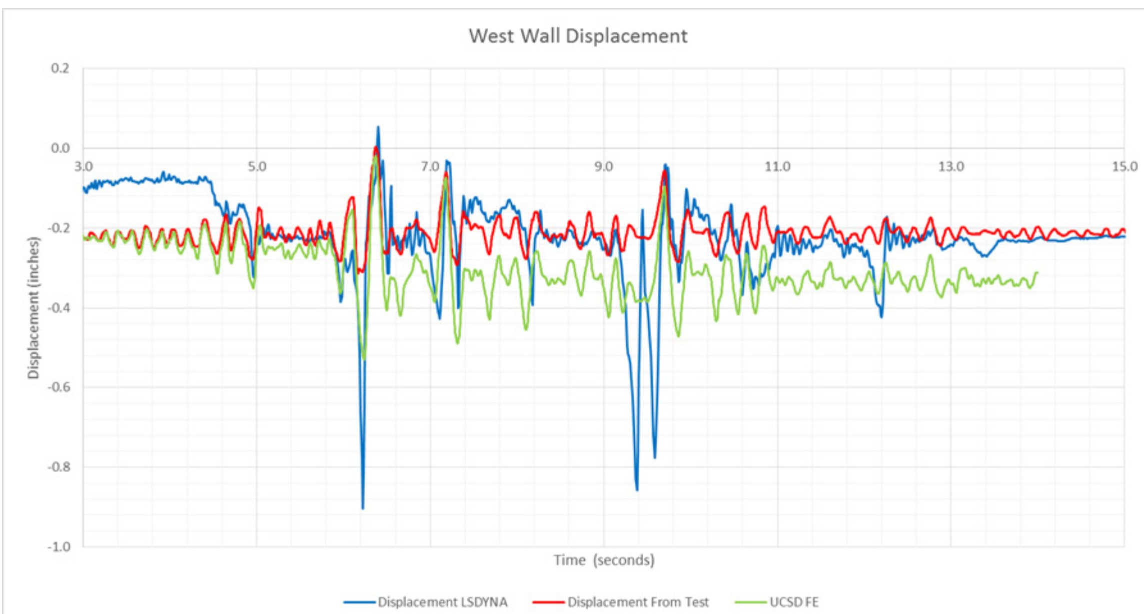

Figure 15. West Wall Deflection

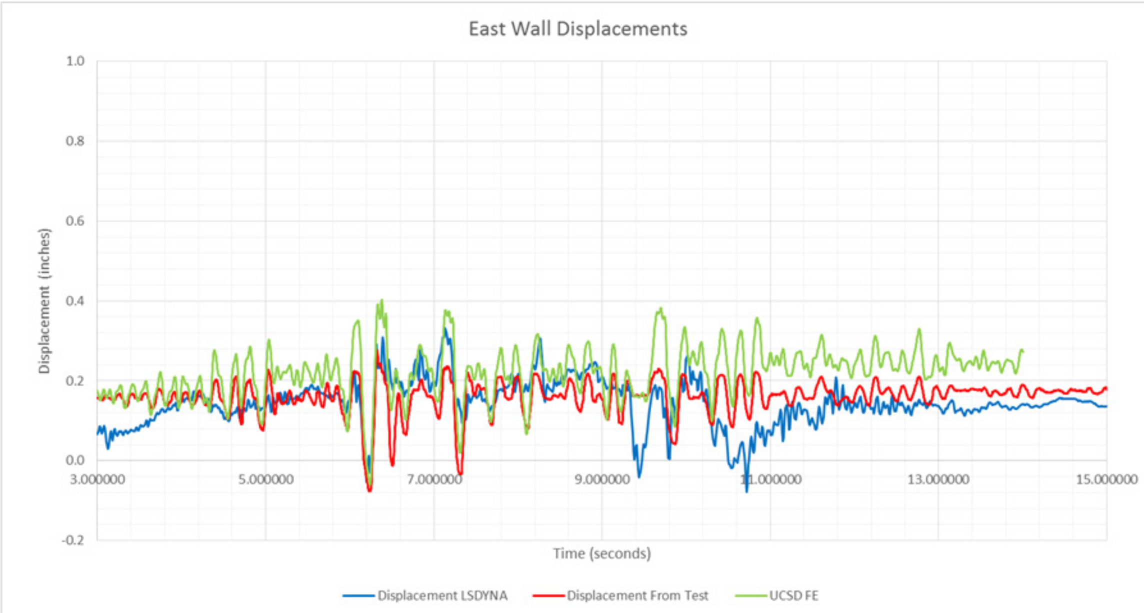

Figure 16. East Wall Deflection

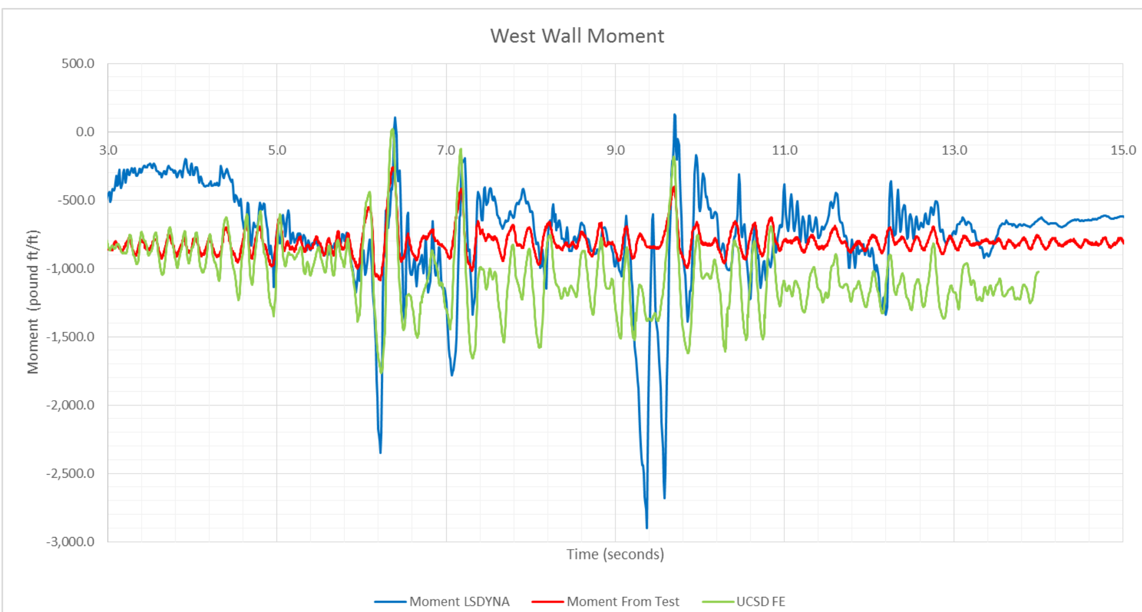

Figure 17. West Wall Base Moment

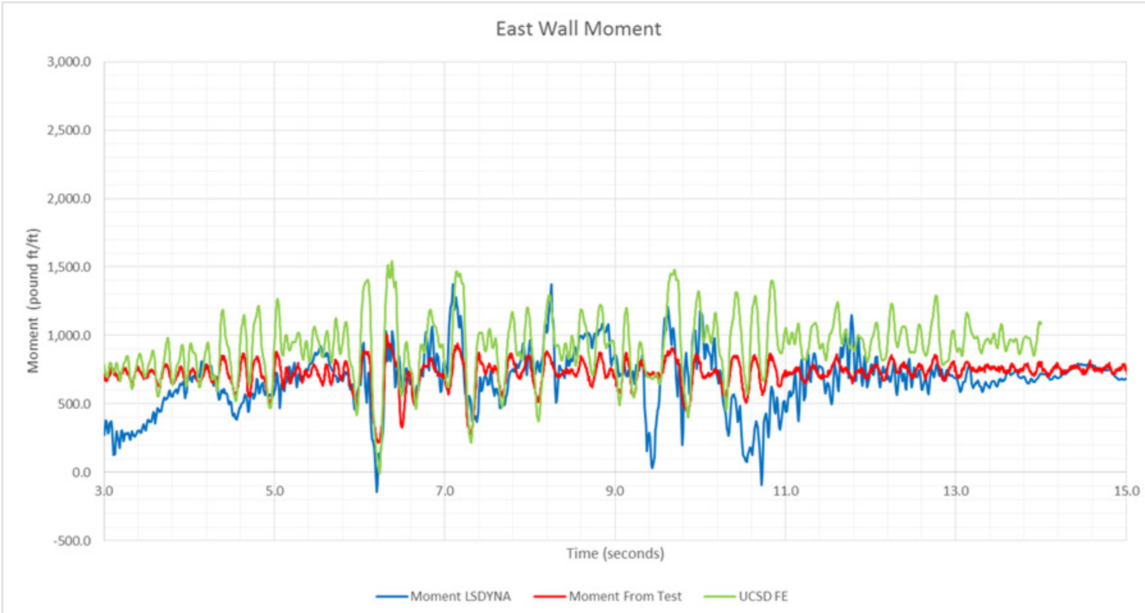

Figure 18. East Wall Base Moment

As can be seen, Reclamation results overestimate the test results for the west wall significantly. The pattern does not match well with the test results, displaying large spikes and high frequency content. The east wall results are similar in terms of high frequency content, however there is a reduction in spikes. The west side soil has a greater phi angle associated with it; 53.2 degrees compared to 40 degrees for the east side soil. A stronger soil would tend to yield less and produce larger reactions against the wall interacting with it; hence the larger moment and displacement spikes.

Both the Reclamation and UCSD models tend to exhibit larger oscillations as compared with the test results. This may indicate that the finite element models trap energy with not enough damping included. The test mechanism itself includes the laminar frames which provide a means by which energy propagating through the soil can be absorbed.

Another underlying concern is that the Reclamation finite element model does not have any compaction effort included, meaning that the soil matrix is less compact.

FE parametric studies

In order to further advance this research effort, the Reclamation finite element model was rerun changing various soil parameters. The following is a comprehensive list of the changes done to date, with results and discussion following. Only some of the parametric studies are discussed in detail because they offered more promising results. The parametric studies included:

1) Using the modified soil properties according to Figure 7.
2) Changing the equation of state
3) Investigating the use of an elastic soil material
4) Changing the contact between the walls and soil
5) Changing the cohesion and phi angles of the soils

The following figures show the results based on some of these changes. In particular, modified soil properties according to Figure 9 and changing the cohesion and phi angle will be discussed. Figure 19 and Figure 20 show the displacements at the top of the west and east walls respectively, while Figure 21 and Figure 22 show the moments at the bottom of the west and east walls respectively.

In each figure there are four sets of results. Once again, the blue line shows Reclamation finite element results and the red line shows data from the shake table test itself. The green line shows Reclamation results using the modified soil properties as shown in Figure 7 while the orange line shows results with a very low phi angle of five degrees and a cohesion of 0.63 psi.

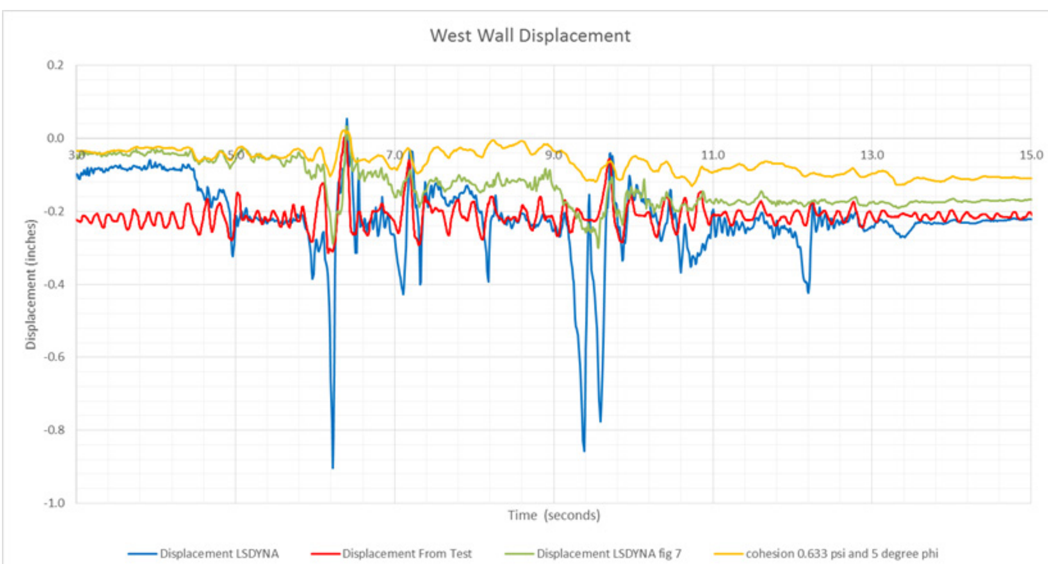

Figure 19. West Wall Deflection

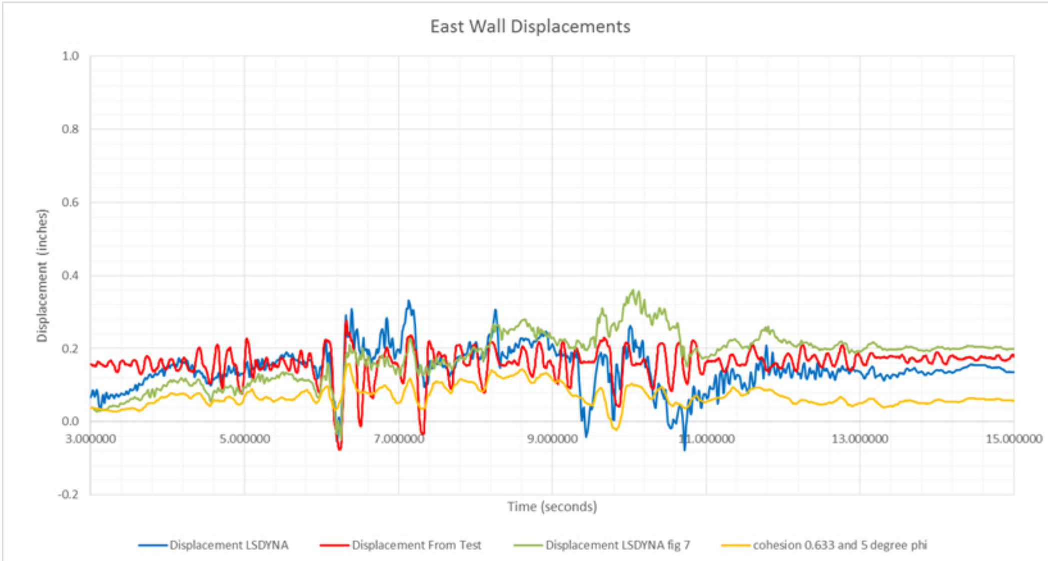

Figure 20. East Wall Deflection

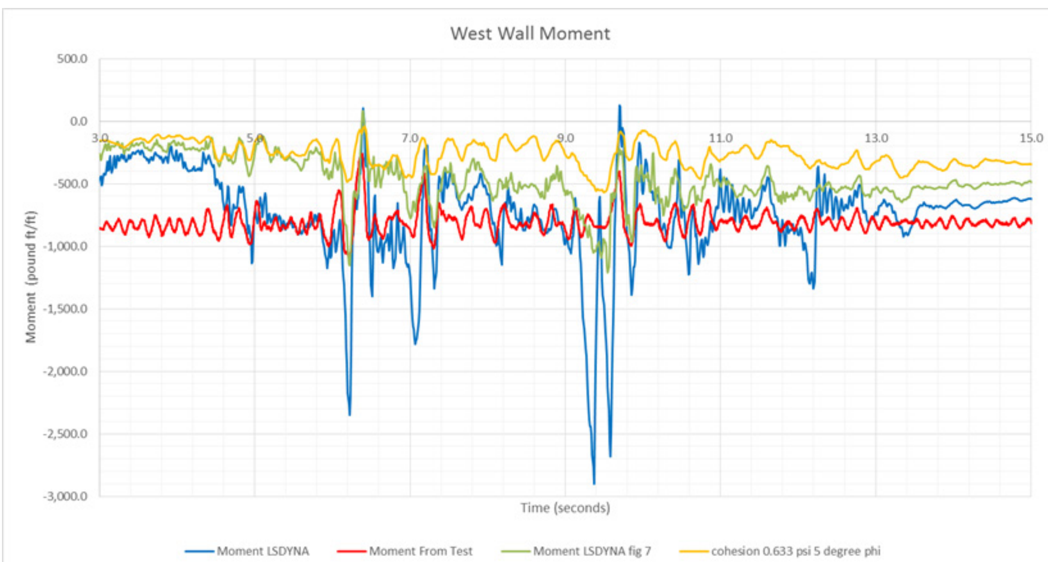

Figure 21. West Wall Base Moment

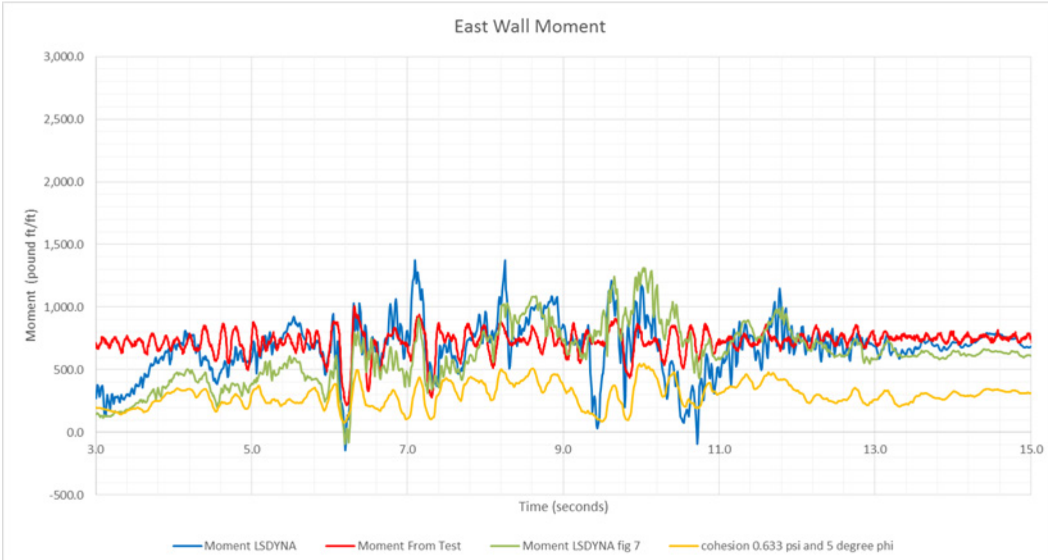

Figure 22. East Wall Base Moment

In general, using the modified soil properties according to Figure 7 reduced the large spikes in moment and displacement as compared with the original Reclamation results. However, there is still considerable high frequency content and the pattern does not match well with the test data. Reducing the phi angle to something small compared with the original data (five degrees picked as a value to use) and using a low cohesion (0.633 psi picked as a value to use) reduced the high frequency content with the patterns now more representative of the test result patterns. However, now the magnitude and the amplitude of the patterns are low as compared with test data. The magnitude of the test data includes large initial values of static moment and static displacement of the walls. This is probably due to the compaction effort applied. At this time, the Reclamation finite element analysis do not include this effort.

Conclusions and Recommendations

Based on the results, the following conclusions can be made:

1) Using the original soil material properties, the Reclamation finite element model predicts both displacements and moments in both the west and east walls that exceed the physical model results.
2) The test data suggests that the soil does not exhibit significant nonlinear behavior since the displacements and moments go back to pre-shaking values. In comparison, the finite element results show nonlinear behavior with post-shaking deflections and moments larger than pre-shaking values.
3) Parametric studies seem to indicate that the use of small phi angles and some cohesion in the soils next to the walls and under the walls tends to reduce the high frequency content of the Reclamation results.
4) A combination of a small phi angle and some cohesion has the potential of representing the behavior of the walls in the test. Although using a small phi angle and some cohesion for sand is not intuitive, it appears that the compaction effort transformed the sand into a soil that can be described by them. More investigation into this is required.
5) Inclusion of a compaction effort may bring the Reclamation results more in line with the test data in terms of magnitude. Compaction efforts can be difficult to model but some means of accounting for them needs to be addressed.

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