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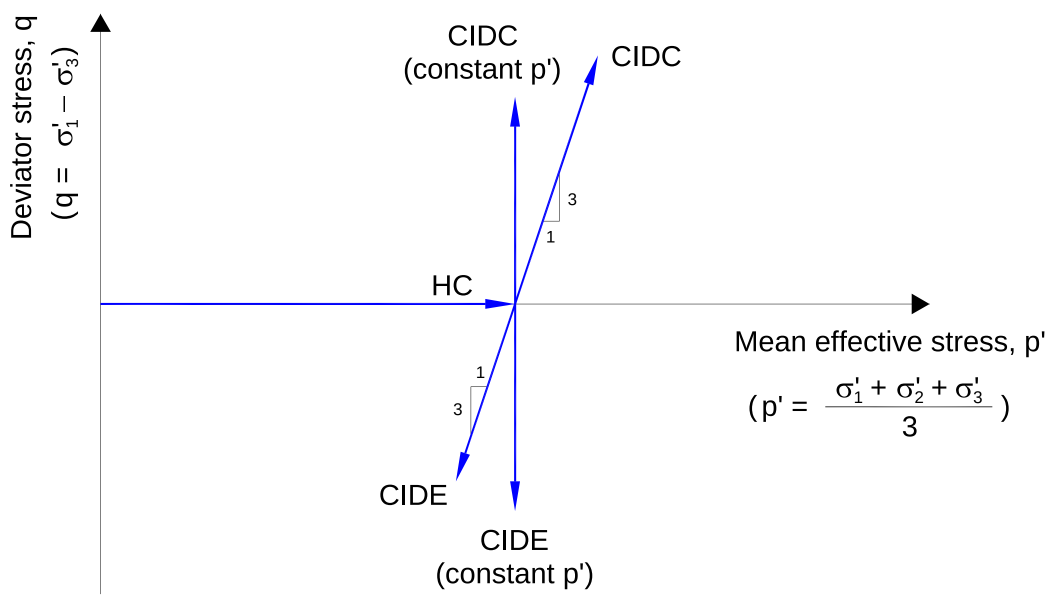
Report DSO-14-02

Verification of FLAC Mohr-Coulomb Model for Granular Materials under Monotonic Loading

Dam Safety Technology Development Program



U.S. Department of the Interior
Bureau of Reclamation
Technical Service Center
Denver, Colorado

August 2014

1. REPORT DATE *(DD-MM-YYYY)*
18-08-2014

2. REPORT TYPE

3. DATES COVERED ***(From - To)***

4. TITLE AND SUBTITLE
Verification of FLAC Mohr-Coulomb Model for Granular Materials under Monotonic Loading

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Bureau of Reclamation, Technical Service Center, Geotechnical Engineering Group 3
PO Box 25007
Denver, CO 80225-0007

8. PERFORMING ORGANIZATION REPORT NUMBER
DSO-14-02

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Bureau of Reclamation, Dam Safety Office
PO Box 25007
Denver, CO 80225-0007

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22161

13. SUPPLEMENTARY NOTE

14. ABSTRACT
The computer program FLAC has been used extensively by Bureau of Reclamation (Reclamation) for determining embankment dam deformations under seismic loads for Dam Safety projects. However, there are substantial uncertainties regarding the predictive capability of FLAC. Verification and validation (V&V) process is needed in order to evaluate the predictive capability of FLAC for embankment dam applications. One of the components in the V&V process for numerical model simulation is to examine material behavior at the element level. At Reclamation, geologic materials are modeled almost exclusively using the FLAC Mohr-Coulomb model for its simplicity, as it only requires four shear strength parameters and two elastic moduli. Without the verification assessment, it is uncertain if FLAC Mohr-Coulomb model could generate stress-strain responses comparable to laboratory element tests. This research study focuses on the verification of FLAC Mohr-Coulomb model subjected to monotonic loading.

15. SUBJECT TERMS
Verification and validation, simulation, granular materials, laboratory stress-strain curves, monotonic loading

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**a. THIS PAGE**			Kevin Zeh-Zon Lee
UL	UL	UL		118	**19b. TELEPHONE NUMBER** *(Include area code)* 303-445-3022

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

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Mission Statements

The U.S. Department of the Interior protects America's natural resources and heritage, honors our cultures and tribal communities, and supplies the energy to power our future.

The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.

BUREAU OF RECLAMATION
Dam Safety Technology Development Program
Geotechnical Engineering Group 3, 86-68313

DSO-14-02

Verification of FLAC Mohr-Coulomb Model for Granular Materials under Monotonic Loading

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REVISIONS					
Date	Description	Prepared	Checked	Technical approval	Peer review

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1. INTRODUCTION

The computer program FLAC has been used extensively by the Bureau of Reclamation (Reclamation) for estimating embankment dam deformations under seismic loads for Dam Safety projects. However, there are uncertainties regarding the predictive capability of FLAC. The verification and validation (V&V) process is needed in order to evaluate the predictive capability of FLAC for embankment dam applications. One of the components in the V&V process for numerical model simulation is to examine material behavior at the element level. At Reclamation, geologic materials are modeled almost exclusively using the FLAC Mohr-Coulomb model for its simplicity, as it only requires four shear strength parameters and two elastic moduli. Without the verification assessment, it is uncertain if FLAC Mohr-Coulomb model could generate stress-strain responses comparable to laboratory element tests. This research study focuses on the verification assessment of FLAC Mohr-Coulomb model subjected to monotonic loading.

The objective of this study is two-fold. The first objective is to evaluate the effectiveness of the FLAC Mohr-Coulomb material model in predicting nonlinear stress-strain response of granular materials under controlled monotonic loading. The FLAC Mohr-Coulomb model was evaluated against laboratory element tests subjected to different stress paths (e.g., hydrostatic compression, conventional triaxial compression, conventional triaxial extension, and direct simple shear). The second objective is to perform a parametric study to evaluate the effects of parameters associated with FLAC Mohr-Coulomb model. Results of this study allows Reclamation analysts to better understand the capabilities and limitations of the Mohr-Coulomb model at the element level.

In this report, the terminology associated with numerical model simulation is first defined. Method of analysis is then discussed, which includes the descriptions of FLAC and the Mohr-Coulomb material model. Following the method of analysis, verification and parametric studies are addressed. Lastly, the conclusions and recommendations for future studies are presented and discussed.

2. TERMINOLOGY

More than ever, numerical simulation of a physical model or process is performed for all disciplines of engineering on a routine basis. The popularity of numerical simulation nowadays can be attributed to easy accessibility of computer programs and the ability to include model details. Numerical simulation fulfills engineering enquiries such as to make quantitative predictions, to compare alternatives, and to identify governing parameters, design limitations, and modes of failure. In addition, numerical simulation draws great attention since it (1) provides in-depth understanding of the physical process, (2) is more economical than a physical model, (3) allows for parametric study, (4) involves no safety concern for personnel performing the simulation, (5) contains known solutions in the domain of interest, and (6) allows for different boundary and initial conditions that are not easily achievable in a physical model (Krahn [1]). It is noted in the geotechnical engineering literature that "verification" and "validation" are sometimes interchangeable. However, particularly in numerical simulation, distinction between the two needs to be made in order to conform to the terminology that is widely used in the simulation community for effective communications. The terms of verification, validation, prediction, and calibration pertaining to numerical simulation are defined in the following sections. It should be noted that only the model verification was involved in this study.

2.1 Verification and Validation

Potential use of the results from numerical simulation, especially for decision making or engineering design, is related to the confidence that one might have in the simulation. An approach to establish the confidence or the credibility of the numerical simulation is through the verification and validation (V&V) process. The American Institute of Aeronautics and Astronautics (AIAA [2]) has defined the verification as "The process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model." AIAA defines the validation as "The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model." Definitions of verification and validation are shown graphically in Figures 2.1 and 2.2, respectively. More concisely, Oberkampf et al. [3] have defined verification as "the assessment of the accuracy of the solution to a computational model" and validation as "the assessment of the accuracy of a computational simulation by comparison with experimental data."

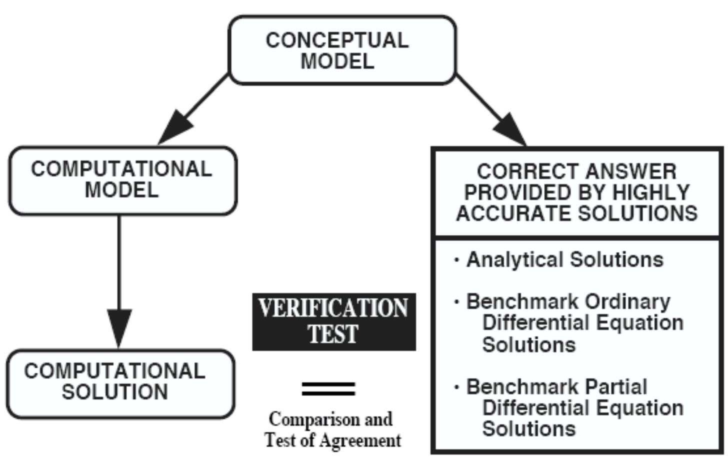


Figure 2.1 – Verification process (after AIAA [2])

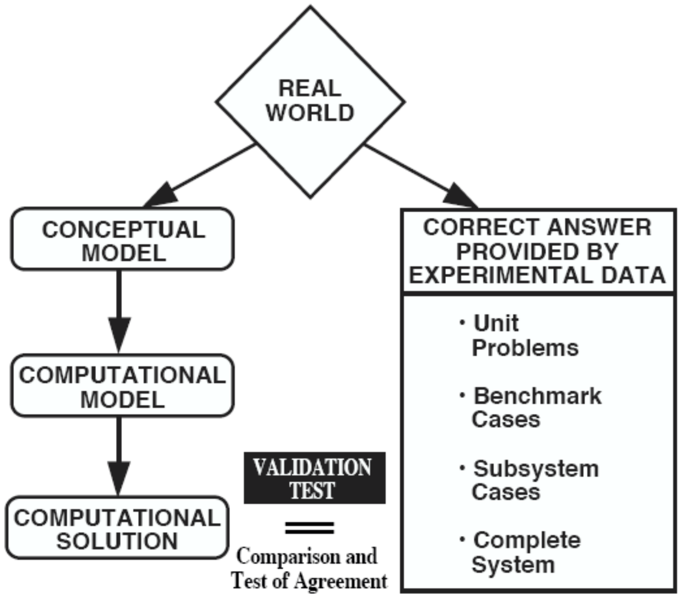


Figure 2.2 – Validation process (after AIAA [2])

The process of V&V proposed by Schlesinger [4] is shown schematically in Figure 2.3, in which the reality can be thought of as the experimental measurements or observed response of a structure. The reality is interrelated to the conceptual model and the computerized model. The conceptual model is composed of mathematical representations, including equations and data, of a physical process of interest. Some examples of conceptual model are: (1) partial differential equations (PDE's) for conservation of mass, momentum, and energy, (2) initial and boundary conditions of the PDE's, and (3) constitutive models for materials (Oberkampf et al. [3]). The computerized model, on the other hand, is a computer program or code compiled based on the conceptual model. As shown in Figure 2.3, verification is performed between the conceptual model and the computerized model, whereas validation relates the outcomes of computerized model to the experimental measurements. Examples of verification activity include numerical algorithm verification, software quality assurance, and numerical error estimation; on the other hand, conducting validation experiments and performing confidence assessment are examples of validation activity (Oberkampf et al. [3]).

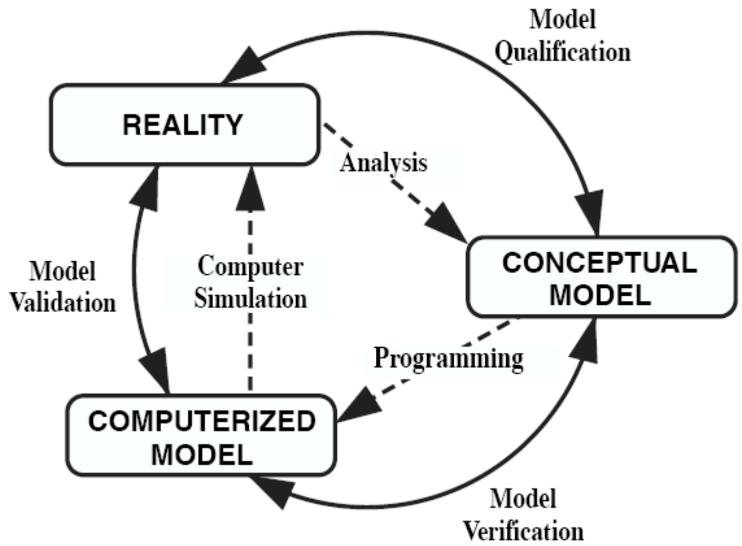


Figure 2.3 – Phases of modeling and simulation (after Schlesinger [4])

V&V process is needed in order for a computer program to establish its predictive capability, and this process requires interaction between experimental and computational activities. The interaction concept has also been recognized in geotechnical engineering as is evident by the soil mechanics triangle illustrated in Figure 2.4, where three aspects of soil mechanics (i.e., ground profile, soil behavior, and modeling) suggested by Burland [5] are interlinked. Note that the soil mechanics triangle concurs with the phases of modeling and simulation as depicted in Figure 2.3 in the way that modeling is interlinked with reality, where reality in geotechnical engineering is represented by soil behavior and ground profile. Burland has emphasized that modeling and reality (e.g., experiments) in the soil mechanics triangle must be linked closely in order to achieve advancement. A meaningful simulation is founded on continuous interactions with the reality, and improvements to a simulation can thus be achieved through continuous cycles of interactions. Note also that the completion or sufficiency of the V&V process is often vaguely defined, since it depends on practical issues such as financial constraints and the intended uses of the model.

Figure 2.1 represents the verification assessment performed in this study. In this study, the FLAC Mohr-Coulomb formulation is treated as the conceptual model. Laboratory measured soil behavior is treated as the real solution to the Mohr-Coulomb formulation. The numerical solutions, in terms of stress-strain responses, are computed by the computer program FLAC. The verification assessment is done by comparing the measured behavior and the calculated responses.

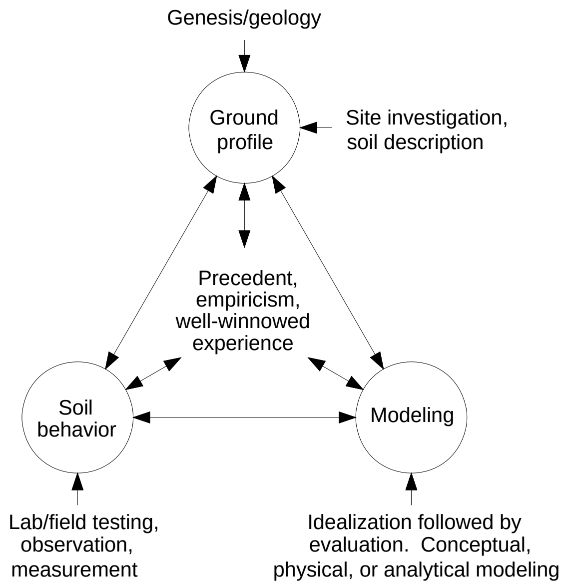


Figure 2.4 – The enhanced soil mechanics triangle (after Anon. [6])

2.2 Prediction

Types of prediction in geotechnical engineering have been defined by Lambe [7] as either being a Class A (before event), Class B (during event), or Class C (after event) prediction. A more stringent definition of prediction pertaining to numerical simulation is given by AIAA [2] as "use of a computational model to foretell the state of a physical system under conditions for which the computational model has not been validated." The prediction definition by AIAA coincides with the Class A prediction suggested by Lambe. The usage of prediction by AIAA is less general as it excludes the precedent comparison of computation results with the experimental data (i.e., validation). As proposed by Oberkampf and Trucano [8], the relationship between validation and prediction is delineated in Figure 2.5, where the prediction is linked by the dashed lines and validation by the solid lines. Note that prediction as defined by AIAA could not indicate the accuracy of a complex system that has not been validated; rather the accuracy can only be inferred based on the previous quantitative comparison.

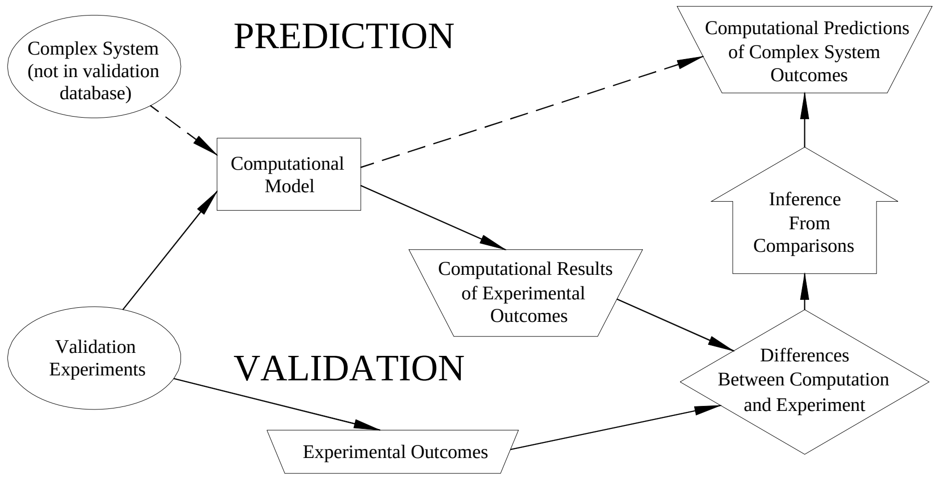


Figure 2.5 – Relationship of validation to prediction (modified from Oberkampf and Trucano [8])

2.3 Calibration

Often times, a great amount of effort could be allocated in model "calibration." As defined by AIAA [2], model calibration is the explicit fine tuning of the unknown parameters in the computational model to achieve some level of agreement with the experimental data. With this definition, model calibration is then equivalent to the Class C prediction suggested by Lambe [7], which is an after-event activity. In addition, calibration allows the identification of those input parameters that could significantly affect the result of numerical simulation. Stated differently, the sensitivity of the parameters is evaluated in the calibration process. A successful calibration could hence establish appropriate values of the parameters when making future prediction. This study, however, does not involve model calibration. Calibration is performed in the validation activity rather than the verification activity.

3. METHOD OF ANALYSIS

The FLAC Mohr-Coulomb model has been used almost exclusively to represent geologic materials in the numerical simulations performed by Reclamation. This section starts with an introduction of the computer program FLAC and is followed by the description of the FLAC Mohr-Coulomb model implemented in FLAC. FLAC Mohr-Coulomb model parameters are also presented.

3.1 Computer Program

The computer program FLAC [9] is a two-dimensional, explicit, finite-difference computer program for engineering mechanics computation. The program has been developed primarily for geotechnical applications. This program can be used to simulate the behavior of structures constructed of soil, rock or other materials that may undergo plastic flow when their yield limits are reached. The materials are discretized by elements (four-node quadrilateral), also known as zones within FLAC, which form a grid that can be shaped by the user to conform to the physical structure being modeled. Each element will behave according to a prescribed linear or nonlinear stress-strain material model in response to the applied forces and boundary conditions.

FLAC uses an explicit, time-marching routine to solve the full equations of motion (including inertial terms). The program permits the analysis of progressive failure and collapse, which may involve local or general instability. The explicit routine allows for soil modeling at, or near, failure more efficiently than an implicit finite element scheme. Even for the static solution, the dynamic equations of motion are included in the formulation. FLAC contains a built-in programming language called FISH (short for FLACish), which enables the user to customize new variables and functions. The user can operate FLAC in menu-driven or command-driven mode and can switch back and forth between the two modes. These capabilities make FLAC a suitable computer program for performing complex dynamic deformation analyses.

In this study, FLAC analysis was performed as a mechanical-only, non-coupled, simulation, which is appropriate for drained monotonic loading. In contrast to a mechanical-only simulation, a coupled simulation would include analysis with both mechanical and groundwater flow options activated. In the mechanical-only simulation, the pore water pressures within the model are function of depth below the water table and remained unchanged during dynamic loading.

3.2 FLAC Mohr-Coulomb Model

The Mohr-Coulomb model is one of the 14 basic constitutive models provided in FLAC, and it belongs to the plastic model group. The models in the plastic model group are characterized by their yield function, hardening/softening functions, and flow rule. In particular, the yield function and flow rule are addressed in the Mohr-Coulomb model, while the hardening/softening functions are not included. The yield function determines the stress condition for which plastic flow takes place. An incremental elastic or plastic behavior is determined by the stress condition below or on the yield surfaces in a generalized stress space, respectively. The main difference between elastic response and plastic response is that plastic flow will be irreversible. The plastic flow formulation in FLAC is based on the assumptions that the total strain increment is the sum of elastic and plastic strains. The elastic strain increment is governed by the elastic relations and stress increment. The implementation of incremental formulation is discussed in detail in the FLAC Constitutive Model Manual [10].

The flow rule is used to describe the deformation following yield; it defines the relationship between the failure envelope and the direction of the plastic strain increment vector. An associated flow rule occurs when the yield function and the plastic potential function coincide, where the plastic potential function is orthogonal to all of the plastic strain increment vectors. For a perfectly plastic material, the normality condition is achieved when the plastic strain increment vector is normal to the yield surface. Note that normality implies a maximum plastic work rate with a convex yield surface.

3.2.1 Yield Function

Following the principal stress ordering convention in FLAC (i.e., $\sigma_1 \leq \sigma_2 \leq \sigma_3$), the Mohr-Coulomb failure envelope in τ-σ space is shown in Figure 3.1. The shear strength (τ_f) of a soil mass at a point on a particular plane is expressed as a linear function of normal stress (σ_f) at the same point on the same plane as:

$$\tau_f = c - \sigma_f \tan\phi \tag{3.1}$$

where c = cohesion and ϕ = friction angle, and c and ϕ are known as the shear strength parameters. From Figure 3.1, the relationship between principal stresses at failure and the shear strength parameters can be obtained as:

$$\sin\phi = \frac{-\frac{1}{2}(\sigma_1 - \sigma_3)}{-\frac{1}{2}(\sigma_1 + \sigma_3) + \dfrac{c}{\tan\phi}} \tag{3.2}$$

therefore

$$\sigma_3 \frac{1+\sin\phi}{2\,c\cos\phi} - \sigma_1 \frac{1-\sin\phi}{2\,c\cos\phi} = 1 \tag{3.3}$$

or

$$\sigma_1 = \sigma_3 \frac{1+\sin\phi}{1-\sin\phi} - 2\,c\,\frac{\cos\phi}{1-\sin\phi} \tag{3.4}$$

Using the identity,

$$\frac{\cos\phi}{1-\sin\phi} = \sqrt{\frac{1+\sin\phi}{1-\sin\phi}} = \sqrt{N_\phi} \tag{3.5}$$

Equation 3.4 can be expressed as:

$$\sigma_1 = \sigma_3\, N_\phi - 2\,c\,\sqrt{N_\phi} \tag{3.6}$$

or

$$f^s = \sigma_1 - \sigma_3 \; N_\phi + 2\,c\,\sqrt{N_\phi} \qquad (3.7)$$

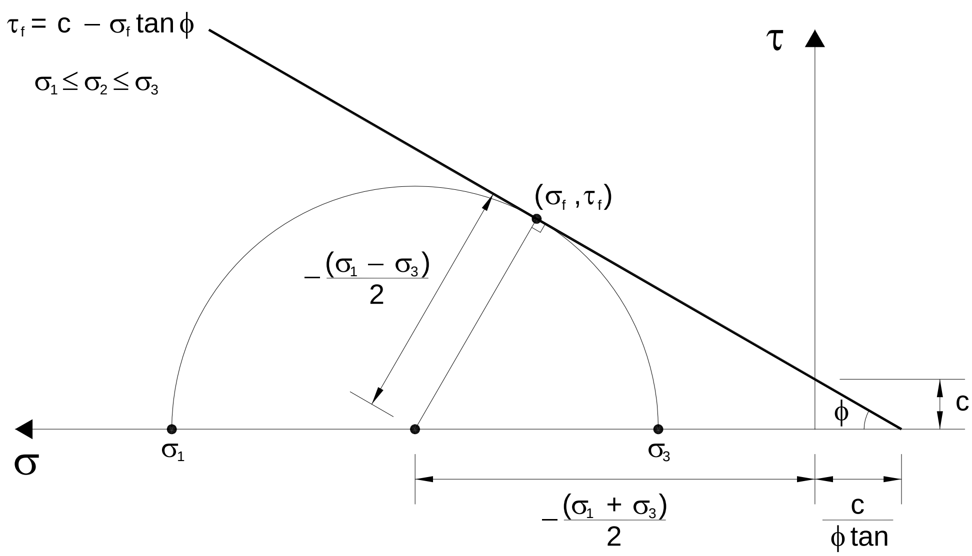


Figure 3.1 – Mohr-Coulomb failure envelope with one Mohr circle at failure (with FLAC sign convention)

The Mohr-Coulomb failure criterion in σ_3-σ_1 space is presented in Figure 3.2. In addition to the shear yield function f^s, the criterion also includes a tension yield function f^t of the form:

$$f^t = \sigma^t - \sigma_3 \qquad (3.8)$$

where σ^t = tensile strength. For a c-ϕ soil, the tensile strength cannot exceed the value σ^t_{max} given by:

$$\sigma^t_{max} = \frac{c}{\tan\phi} \qquad (3.9)$$

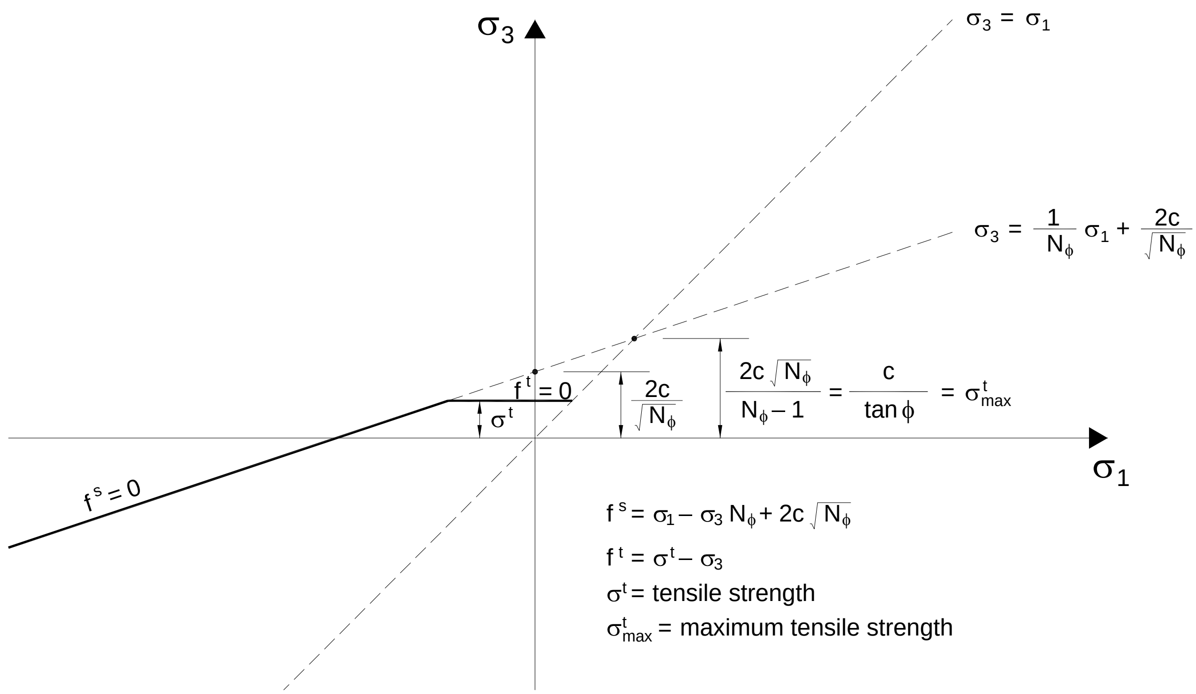


Figure 3.2 – Mohr-Coulomb failure criterion in σ_3-σ_1 space

It should be noted that the sign conventions adopted by FLAC follow those from structural engineering, which are opposite from the sign conventions of soil mechanics. In FLAC, positive stresses and strains indicate tension, and negative stresses and strains indicate compression. The equations presented above are based on the FLAC sign conventions. To be consistent with geotechnical laboratory data presentation, soil mechanics sign conventions were used in Verification Study and Parametric Study sections (Sections 4 and 5) of this report, where compressive stresses and strains are considered positive.

3.2.2 Plastic Potential Function

The FLAC Mohr-Coulomb model implements the nonassociated flow rule for the shear potential function g^s as:

$$g^s = \sigma_1 - \sigma_3 N_\psi \qquad (3.10)$$

where ψ is the dilation angle and N_ψ is defined as:

$$N_\psi = \frac{1+\sin\psi}{1-\sin\psi} \qquad (3.11)$$

On the other hand, the associated flow rule is implemented for the tension potential function g^t as:

$$g^t = -\sigma_3 \tag{3.12}$$

The flow rule at the shear-tension edge is also defined. The shear-tension edge is shown in Figure 3.3. The edge function h, between $f^s = 0$ and $f^t = 0$, has the form:

$$h = \sigma_3 - \sigma^t + \alpha^p \left(\sigma_1 - \sigma^p\right) \tag{3.13}$$

where constants α^p and σ^p are defined as:

$$\alpha^p = \sqrt{1 + N_\phi^{\ 2}} + N_\phi \tag{3.14}$$

and

$$\sigma^p = \sigma^t \ N_\phi - 2\,c\,\sqrt{N_\phi} \tag{3.15}$$

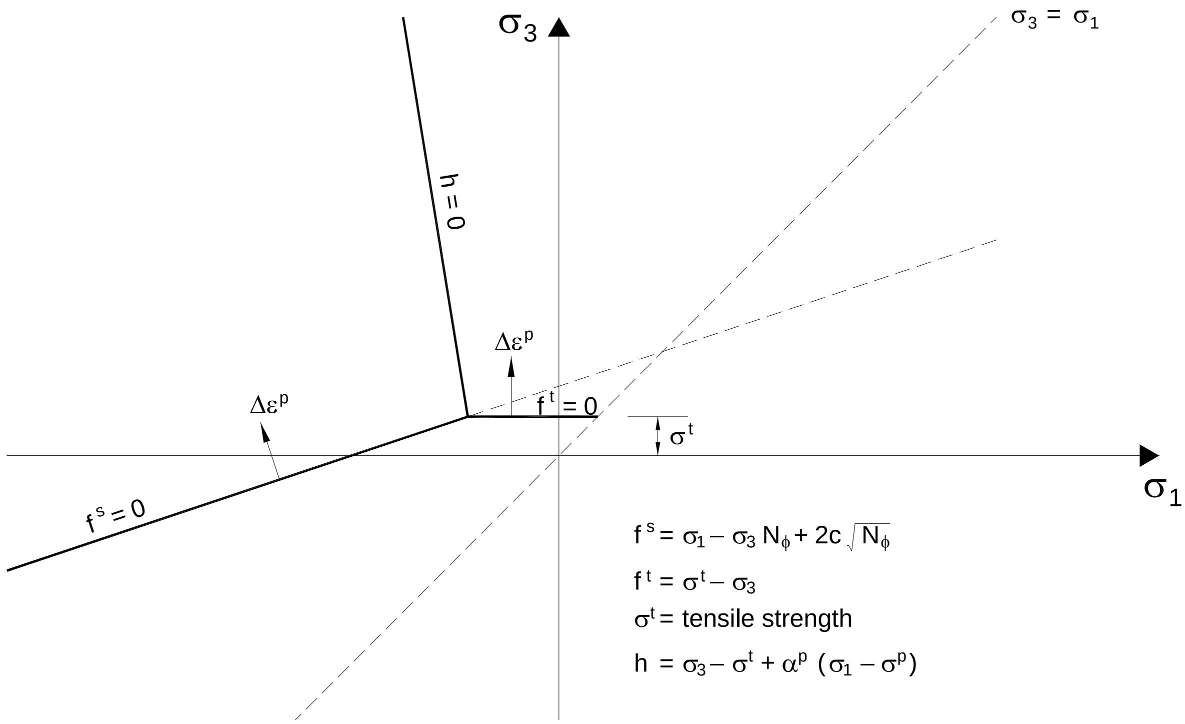


Figure 3.3 – Flow rules of Mohr-Coulomb model in σ_3-σ_1 space

3.2.3 Model Parameters

The seven material parameters for the FLAC Mohr-Coulomb model include:

ρ mass density (10^3 kg/m^3)

K	elastic bulk modulus (kPa)
G	elastic shear modulus (kPa)
c	cohesion (kPa)
ϕ	friction angle (degree)
ψ	dilation angle (degree)
σ^t	tension limit (kPa)

Since the sands examined in this study are cohesionless, the cohesion (c) and tension limit (σ^t) were set to zero. In addition, since gravity was not applied in the single-element model test, the value of mass density (ρ) would not affect the calculated stress-strain responses. However, ρ was entered in the input file to allow for identification of the soil being analyzed. The mass density was also used in determining the elastic moduli.

3.2.3.1 Elastic Properties

Current Reclamation practice for finding the elastic moduli is to use results of low-strain geophysical tests such as shear wave seismic refraction tests or crosshole shear-wave tests. The measured shear wave velocity is in turn used to calculate the shear modulus (G) of the material as:

$$G = \rho \, V_s^{\,2} \qquad (3.16)$$

where ρ = mass density and V_s = shear wave velocity. It should be noted that the mass density in Equation 3.16 has not been defined explicitly in the literature for application in soil mechanics. The mass density could be total (or moist) density, dry density, saturated density, or buoyant (or submerged) density. It is known that the shear modulus depends on void ratio (e) and mean effective stress (p') (Hardin and Black [11]). Intuition also suggests that the shear modulus of sand should be independent of degree of saturation as water has no shearing resistance. Furthermore, the shear wave velocity is independent of the degree of saturation according to Mitchell and Soga [12]. With both G and V_s being independent of the degree of saturation, one would think ρ should be independent of the degree of saturation as well; this, however, has not been proven. In terms of magnitude of G from Equation 3.16, use of saturated density would yield the highest G, while use of buoyant density would result in the lowest G. In this study, dry density was used to estimate G, since dry density is approximately the average value between saturated and buoyant densities. It should be note that since soil behavior is nonlinear, the precise determination of G would be extremely difficult. The bulk modulus (K) can then be approximated from a known G as:

$$K = \frac{2\,G\,(1+\nu)}{3\,(1-2\,\nu)} \qquad (3.17)$$

where ν = Poisson's ratio. Poisson's ratio under drained condition is typically assumed to be 0.35, which is the value used in this study.

As indicated by Equation 3.16, the shear modulus is also a function of shear wave velocity (V_s). Since the shear wave velocities of the sands selected for this study were not reported in the literature, empirical relations were used to estimate the shear wave velocity. First, the friction angle (ϕ') was used to estimate the Standard Penetration Test (SPT) blow count corrected to one atmosphere pressure of overburden (i.e., 101 kPa or 1 tsf) and 60% of the theoretical free-fall hammer energy, $(N_1)_{60}$. The correlation shown in Figure 3.4 between ϕ' and $(N_1)_{60}$ for fine grained sands was used in this study. Based on $(N_1)_{60}$, the shear wave velocity was then determined from the empirical equation proposed by Tsiambaos and Sabatakakis [13] as:

$$V_s = 92\,(N_1)_{60}^{\;0.341} \tag{3.18}$$

Equation 3.18 is applicable for sandy soils, and V_s is expressed in m/s. Consequently, high values of ϕ' would result in high values of elastic moduli.

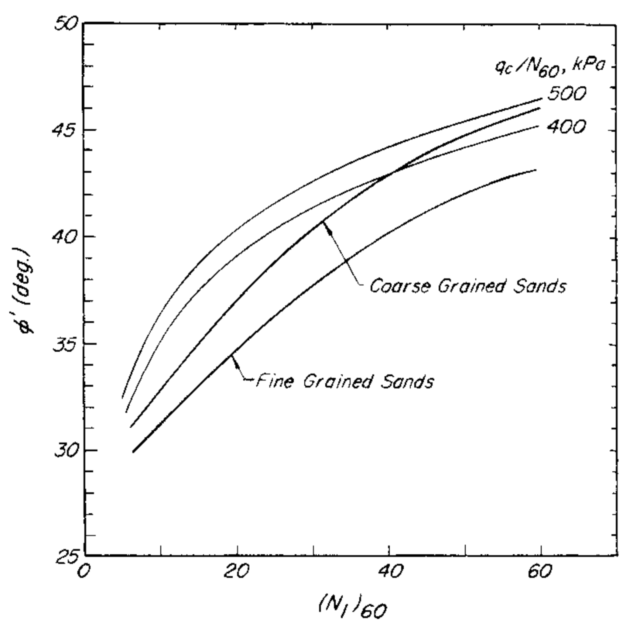


Figure 3.4 – Variation of friction angle ϕ' with corrected SPT blow count $(N_1)_{60}$ (after Terzaghi et al. [14])

3.2.3.2 Friction Angle and Dilation Angle

For this study, failure is said to have occurred when the maximum effective principal stress ratio $(\sigma'_1/\sigma'_3)_f$ is attained. As shown conceptually in Figure 3.5, friction angle varies with relative density and strain level. The peak friction angle

(ϕ'_p) is associated with the maximum effective principal stress ratio $(\sigma'_1/\sigma'_3)_f$. For a dense sand (or dilative soil), the peak friction angle is high, and it develops at small strains. For a loose sand (or contractive soil), the peak friction angle is low and is reached at large strains. For the dense sand, strain softening is observed following the peak friction angle until the critical void ratio state is reached. On the contrary, the loose sand strain-hardens to the critical void ratio state. In theory, a given sand, regardless of the initial density, would converge to the critical void ratio state and exhibit the critical state friction angle (ϕ'_{cv}) at large strains. For cohesionless soil, the residual state friction angle (ϕ'_r) is essentially equal to ϕ'_{cv} (Kulhawy and Mayne [15]).

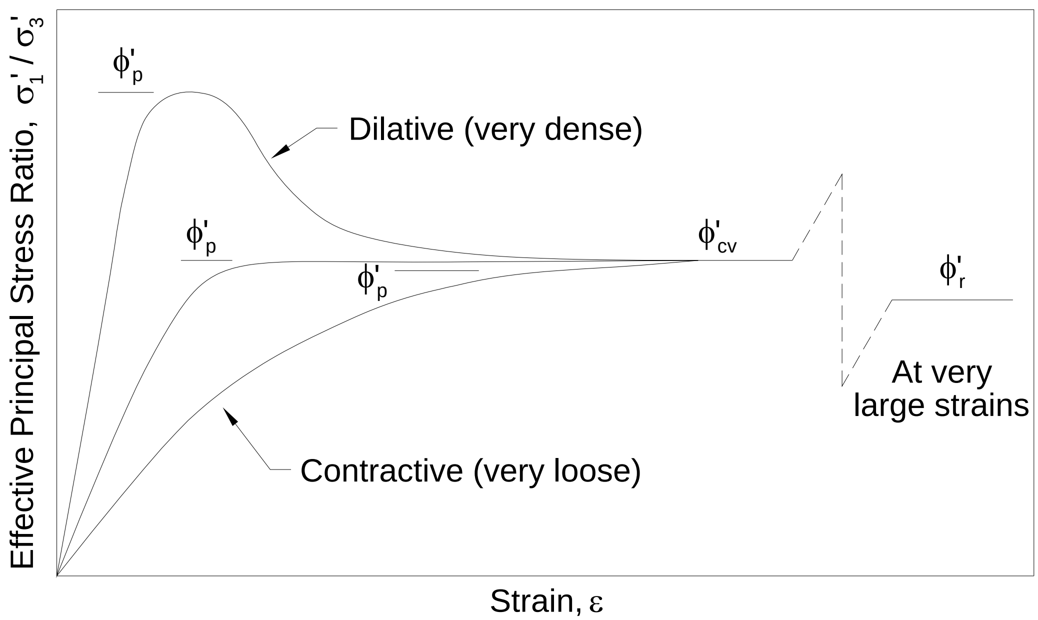


Figure 3.5 – Friction angle definitions (after Kulhawy and Mayne [15])

In laboratory shear strength tests, soil specimens are typically prepared at a target relative density (D_r) and are then sheared under different confining stresses. With failure defined as the maximum effective principal stress ratio, Mohr circles at failure can be constructed (i.e., with known σ'_{1f} and σ'_{3f}). A curved failure envelope is generally observed for granular material due to particle breakage effects (Duncan and Wright [16]). However, for simplicity, a linear Mohr-Coulomb failure envelope was assumed in this study based on Mohr circles at failure with the zero cohesion assumption. The ϕ'_p of the linear failure envelope was selected as the friction angle for the soil in question at a particular relative density. The relationship between friction angle and dilation angle is further discussed in the Dilation Angle section (Section 5.1).

The friction angle from the isotropically-consolidated drained compression (CIDC) tests is considered as the "standard reference" (Kulhawy and Mayne, [15]) in routine design, since other non-conventional tests might be cost prohibitive. Similarly, in current Reclamation practice, a single friction angle is generally used to represent a particular material in an embankment dam model regardless of the stress level and loading condition. As such, the friction angle determined from CIDC tests was used in the single-element model test irrespective of the loading condition.

4. VERIFICATION STUDY

As has been discussed in the Verification and Validation section (Section 2.1), constitutive modeling is considered as part of the verification activity. A conceptual model, such as the Mohr-Coulomb model, was coded in FLAC to calculate stress-strain behavior when the material is subjected to external loads. The purpose of this study is to evaluate the accuracy of the solution of the Mohr-Coulomb model against soil behavior measured from drained monotonic laboratory tests. Laboratory results are regarded as the real solutions similar to the solutions of a PDE problem. The following sections present the index properties of the sands, laboratory shear strength tests, and results of the single-element model tests.

4.1 Descriptions of Soils and Laboratory Tests

The three granular soils examined in this study are Sacramento River Sand, Hostun Sand, and Nevada Sand. The laboratory stress-strain data were obtained from test results published in the literature. The sands were subjected to drained monotonic loading conditions. Types of test considered are summarized in Table 4.1. The stress paths associated with various triaxial tests in the q-p' space are shown in Figure 4.1, in which q is the deviator stress and p' is the mean effective stress (or effective confining stress) defined as:

$$q = \sigma'_1 - \sigma'_3 = \sigma'_a - \sigma'_r \tag{4.1}$$

$$p' = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} = \frac{\sigma'_a + 2\sigma'_r}{3} \tag{4.2}$$

where σ'_1 and σ'_3 are major and minor principal stresses, respectively; σ'_a and σ'_r are axial and radial stresses, respectively. Types of laboratory test relevant to field conditions in an embankment foundation failure are shown schematically in Figure 4.2. Note that the purpose of conducting CIDC and CIDE tests with constant p' during shearing is to evaluate the shear induced dilatancy. In this case, volumetric change due to mean effective stress is eliminated, and the volume change is due solely to deviator stress.

Table 4.1 – Nomenclature used for designation of laboratory tests

Nomenclature	Description
CIDC	Isotropically-consolidated drained compression test (axial compression).
CIDC (p' = constant)	Isotropically-consolidated drained compression test (axial compression). Effective confining stress (p') maintained constant during shear.
CIDE	Isotropically-consolidated drained extension test (axial extension).
CIDE (p' = constant)	Isotropically-consolidated drained extension test (axial extension). Effective confining stress (p') maintained constant during shear.
HC	Hydrostatic (or isotropic) compression test.
DSS	Direct simple shear test.

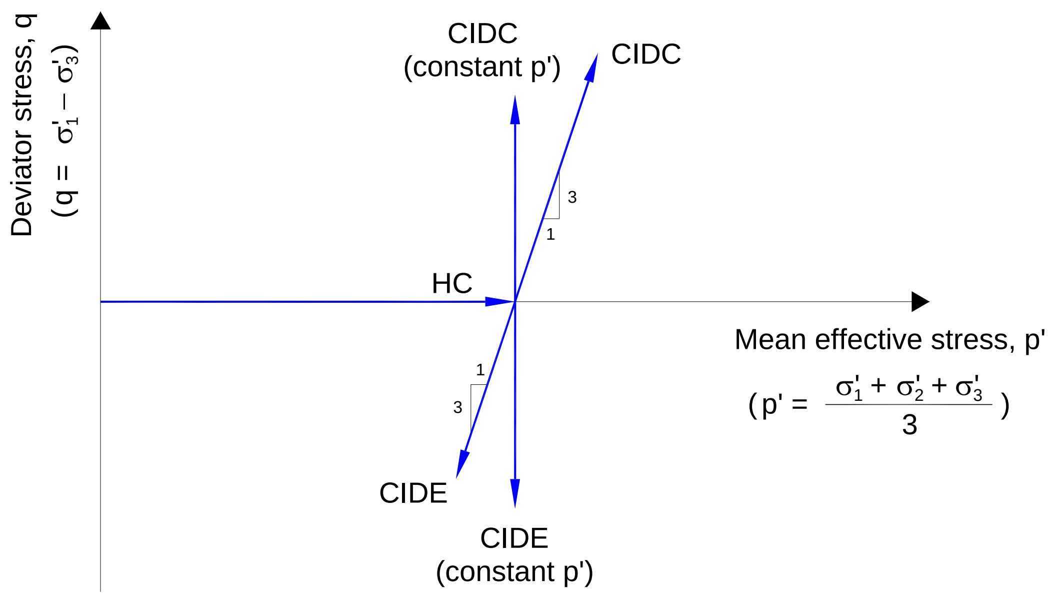


Figure 4.1 – Stress paths for various monotonic triaxial tests

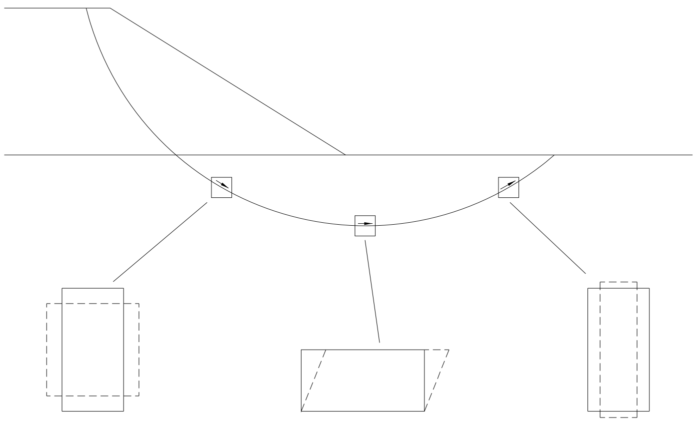


Figure 4.2 – Relevance of laboratory strength tests to field conditions (from Kulhawy and Mayne, [15])

The grain size distributions of the three sands are shown in Figure 4.3. All three sands are uniformly graded fine sands with a Unified Soil Classification System (USCS) of SP. The index properties of the three sands are summarized in Table 4.2. Also included in Table 4.2 are the references of the published laboratory test data. The three sands were selected for their completeness of laboratory test data.

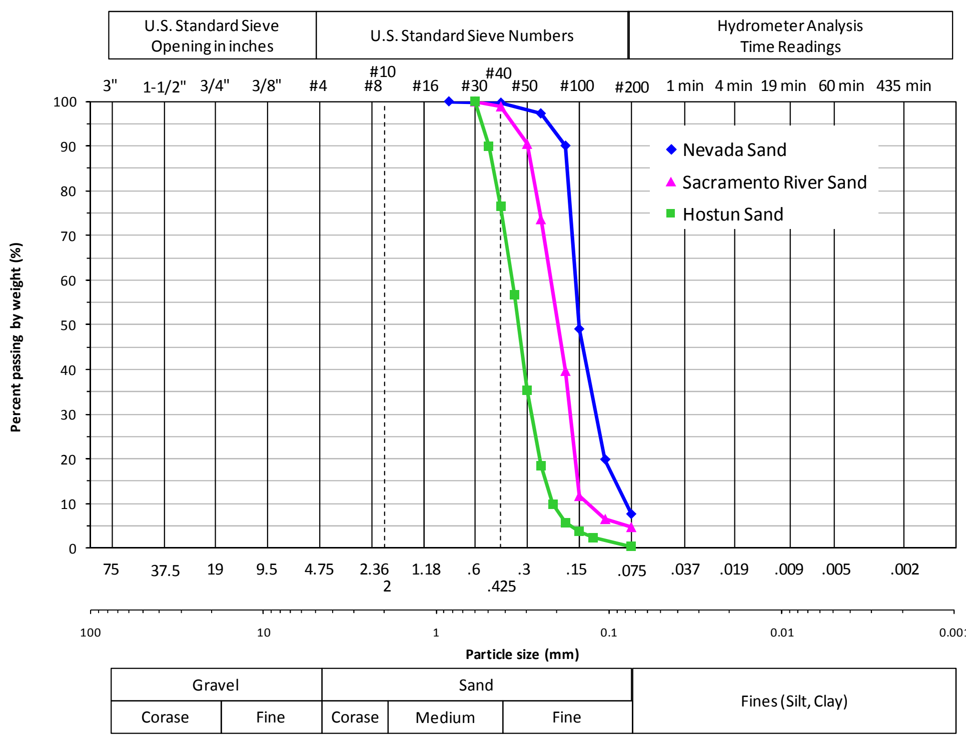


Figure 4.3 – Grain size distribution curves of the three sands evaluated in this study

Table 4.2 – Index properties of the three sands evaluated in this study

Sand	Sacramento River Sand	Hostun Sand	Nevada Sand
USCS	SP	SP	SP
Specific gravity, G_s	2.68	2.667	2.67
e_{min}, e_{max}	0.61, 1.03	0.587, 0.966	0.511, 0.887
ρ_{dmax}, ρ_{dmin} (g/cm^3)	1.665, 1.409	1.681, 1.357	1.767, 1.415
D_{50} (mm)	0.199	0.337	0.151
Coe. of uniformity, C_u	1.64	1.72	1.97
Coe. of curvature, C_c	0.98	1.03	1.13
References	[17], [18]	[19], [20]	[21], [22]

The most significant index parameter for describing the behaviors of cohesionless soil is relative density (D_r), which is a degree of compactness relative to both the loosest and densest states. The relative density can be determined in terms of void ratio (e) as:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \tag{4.3}$$

where e_{max} = maximum void ratio (loosest) and e_{min} = minimum void ratio (densest). D_r can also be determined in terms of dry density (ρ_d) as:

$$D_r = \frac{\rho_{d\,max}(\rho_d - \rho_{d\,min})}{\rho_d(\rho_{d\,max} - \rho_{d\,min})} \tag{4.4}$$

where ρ_{dmax} = maximum dry density (densest) and ρ_{dmin} = minimum dry density (loosest). Higher D_r is generally associated with increasing strength and decreasing compressibility, and the reverse is true for lower D_r. A useful phase relation for finding the dry density (ρ_d) is:

$$\rho_d = \frac{G_s\ \rho_w}{1 + e} \tag{4.5}$$

where, G_s = specific gravity, ρ_w = density of water, and e = void ratio in question. Furthermore, the saturated density (ρ_{sat}) can be determined as:

$$\rho_{sat} = \frac{(G_s + e)\,\rho_w}{1 + e} \tag{4.6}$$

It should be noted that the applicability of D_r should be limited to cohesionless soil with less than 15 percent of fines (Kulhawy and Mayne [15]). In addition, sampling of clean granular soil, especially loose material, is very difficult due to disturbance; hence the exact value of D_r will be uncertain.

4.2 Single-Element Model Test

An axisymmetric condition was applied for all tests considered in this study, except the direct simple shear (DSS) tests. When the model is configured as axisymmetric in FLAC, a cylindrical coordinate system is invoked, where the positive x-direction corresponds to the radial coordinate and the y-direction corresponds to the axial coordinate. The out-of-plane coordinate (z-direction) is the circumferential coordinate. The FLAC axisymmetric grid is viewed as an infinitesimal thin wedge that is constrained from displacement in the circumferential direction. The stress and strain components for an axisymmetric analysis are shown in Figure 4.4. The DSS tests, on the other hand, follow the plane-strain condition, which is the default FLAC model condition.

All single-element model tests were analyzed under strain-control boundary conditions. Strain-controlled tests are better suited for determining the collapse loads than the stress-controlled tests. It is expected that the numerical model would become unstable if stress-control is used to find the collapse load, which is also true for a physical model. In fact, stress-control boundary conditions could not be applied in FLAC for the single-element model test; the calculated stress path under stress-control does not match the stress path inherent to the laboratory test.

It should be noted that the purpose of a single-element model test is to bring forth the solution of FLAC Mohr-Coulomb model. The single-element model test does not include boundary conditions encountered during the actual laboratory test, such as end friction between soil specimen and loading platens. This study assumes the measured stress-strain behavior as the solution to a constitutive model and is representative of a soil element. By utilizing the single-element model, other functions implemented in FLAC not related to constitutive modeling can be eliminated.

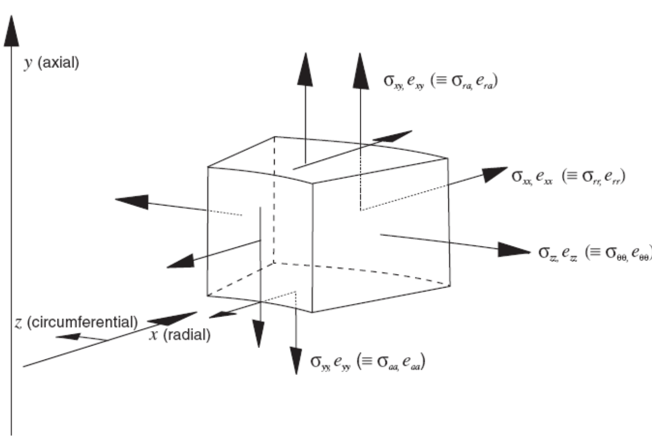


Figure 4.4 – Stress and strain components in FLAC axisymmetric model (after FLAC User's Guide [23])

4.3 Analysis Results

Comparisons of measured and calculated stress-strain responses of the three sands are presented in the following sections. The differences between the measured and calculated responses are noted. Table 4.3 summarizes the test types and Mohr-Coulomb model parameters specified in the analysis. Also included in Table 4.3 are figure numbers corresponding to various tests. The conventional soil mechanics sign convention for stresses and strains are used for result presentations (i.e., compressive stresses and strains are considered positive). The FLAC input files of the tests are included in the Appendix.

4.3.1 Sacramento River Sand

The two tests analyzed for Sacramento River Sand were HC and CIDC tests. The two relative densities considered were 38% and 100%. The peak friction angles from Mohr circles at failure for the two relative densities are shown in Figure 4.5. Note that the linear failure envelopes fit well with Mohr circles at failure for an effective normal stress less than about 1 MPa (20,885.4 lb/ft^2 or 10.4 ton/ft^2). The linear failure envelope over predicts the strength when the effective normal stress exceeds about 1 MPa.

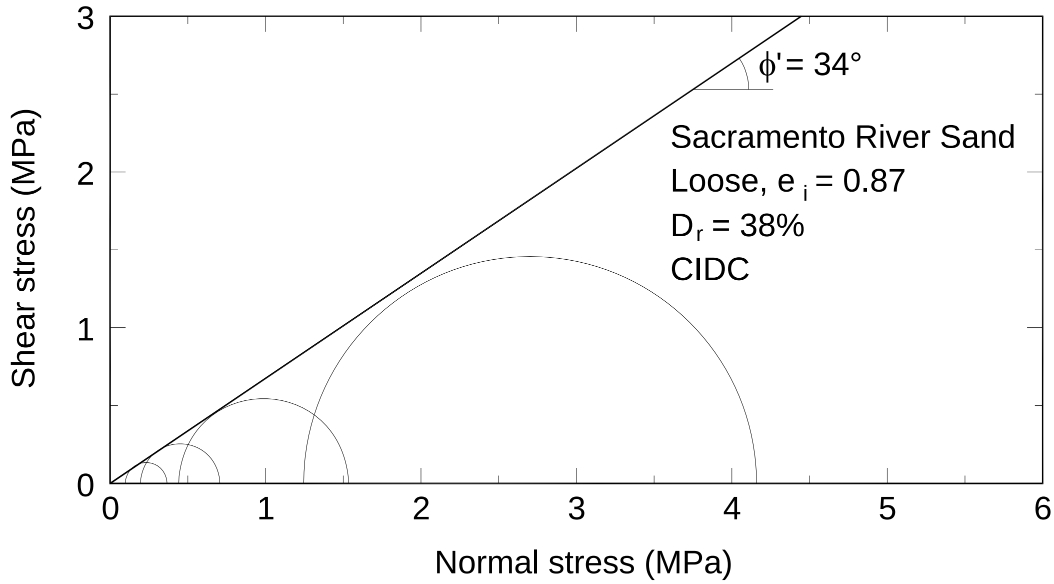
The stress-strain curves of HC tests are compared in Figure 4.6. The results indicate that the calculated stress-strain curves are linear, whereas the measured stress-strain curves are nonlinear. A much stiffer response is observed in the calculated curves than the measured curves. In fact, the slope of the calculated curve is the bulk modulus (K) specified. The hysteretic loops are not observed in the calculated response during loading and unloading cycles.

The stress-strain curves of CDIC tests are compared in Figures 4.7 and 4.8 for D_r = 38% and D_r = 100%, respectively. The differences in measured and calculated stress-strain responses include:

- Nonlinear soil behavior is not captured by FLAC Mohr-Coulomb model.
- The elastic-perfectly plastic stress-strain response is implemented in the FLAC Mohr-Coulomb model.
- Strain softening is not captured by the FLAC Mohr-Coulomb model for a dense sand.
- Failure occurs at a much lower strain level in the calculated response than the measured behavior.
- Soil dilatancy is not captured by the FLAC Mohr-Coulomb model when dilation angle is set to zero. Effects of dilation angle are discussed in Dilation Angle section (Section 5.1).
- Volumetric strain is larger in the measured behavior than the calculated response; FLAC Mohr-Coulomb model under-predicts the volumetric strain.

Table 4.3 – Summary of test types and material model parameters

Sand	Test	D_r (%)	ρ_{sat} (kg/m^3)	ρ_d (kg/m^3)	ϕ' (deg)	V_s (m/s)	G (kN/m^2)	K (kN/m^2)	Fig. No.
Sacramento River	HC	38	1898.4	1433.2	34	246.8	87270.8	261812.3	4.6
Sacramento River	HC	100	2043.5	1664.6	40.4	327.4	178395.2	535185.5	4.6
Sacramento River	CIDC	38	1898.4	1433.2	34	246.8	87270.8	261812.3	4.7
Sacramento River	CIDC	100	2043.5	1664.6	40.4	327.4	178395.2	535185.5	4.8
Hostun	CIDC and CIDE	65.3	1969.8	1551.5	36	272.8	115469.1	346407.2	4.14
Hostun	CIDC and CIDE	92.4	2032.2	1651.4	38.1	298.8	147417.4	442252.3	4.15
Hostun	CIDC and CIDE (p' = constant)	63.7	1966.9	1547.0	35.9	271.5	114048.0	342144.1	4.16
Hostun	CIDC and CIDE (p' = constant)	92.7	2032.8	1652.4	40.3	326.1	175674.4	527023.1	4.17
Nevada	Oedometer	40	1961.4	1537.1	34.4	252.3	97839.1	293517.4	4.20
Nevada	Oedometer	60	2005.4	1607.5	39.1	310.8	155240.1	465720.4	4.20
Nevada	CIDC and CIDE (p' = constant)	40	1961.4	1537.1	34.4	252.3	97839.1	293517.4	4.21
Nevada	CIDC and CIDE (p' = constant)	60	2005.4	1607.5	39.1	310.8	155240.1	465720.4	4.22
Nevada	DSS (constant ht.)	40	1961.4	1537.1	34.4	252.3	97839.1	293517.4	4.23
Nevada	DSS (constant ht.)	60	2005.4	1607.5	39.1	310.8	155240.1	465720.4	4.24



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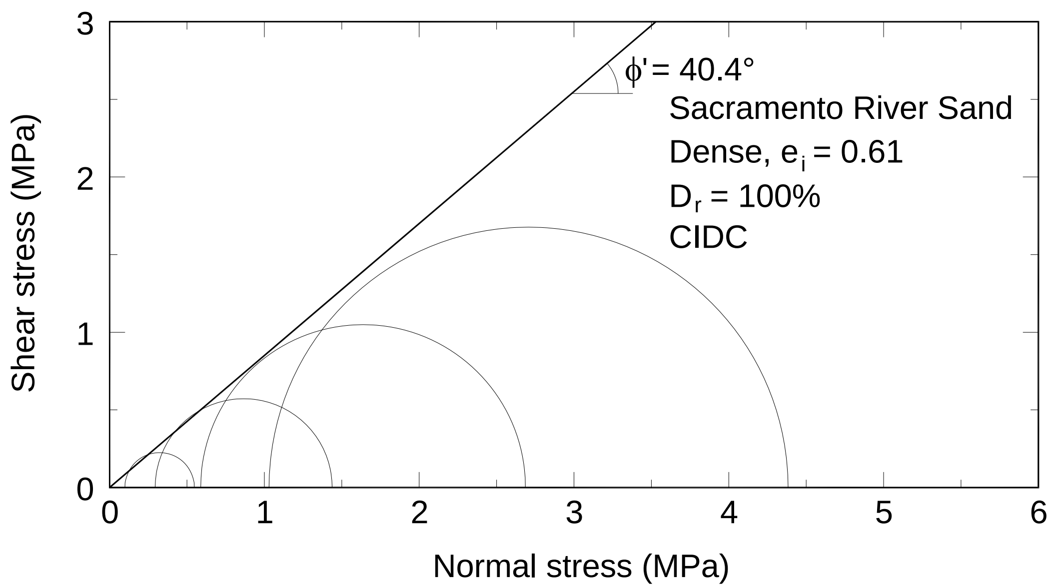
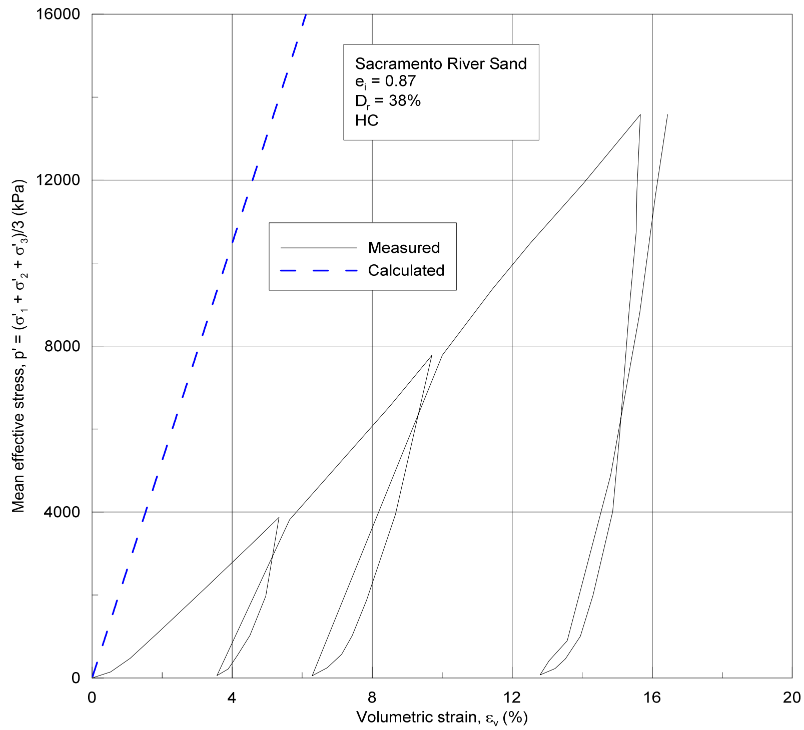
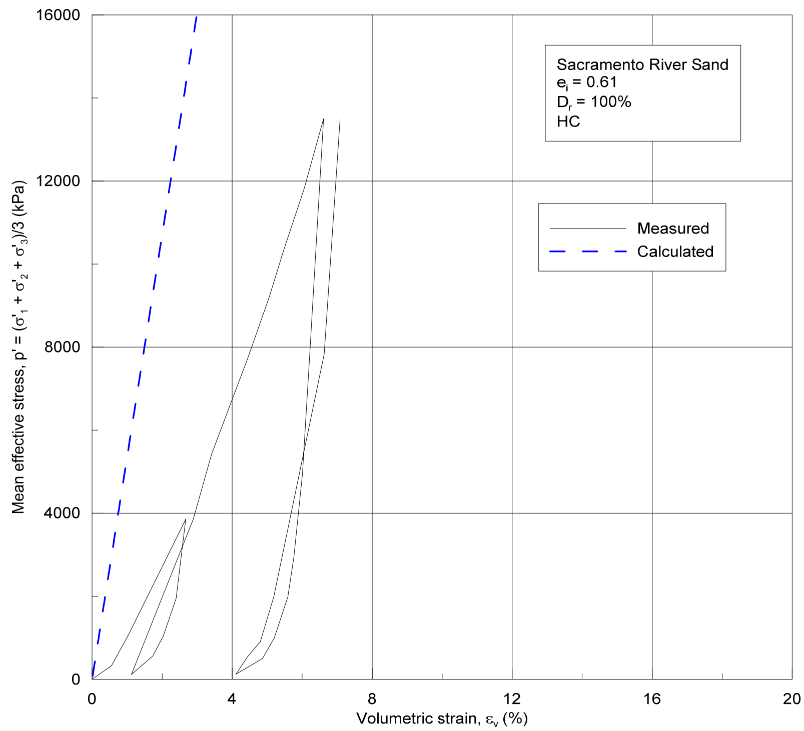


(b)

Figure 4.5 – Mohr circles at failure of Sacramento River Sand for (a) D_r = 38% and (b) D_r = 100%



(a)



(b)

Figure 4.6 – HC stress-strain curves of Sacramento River Sand at (a) D_r = 38% and (b) D_r = 100%

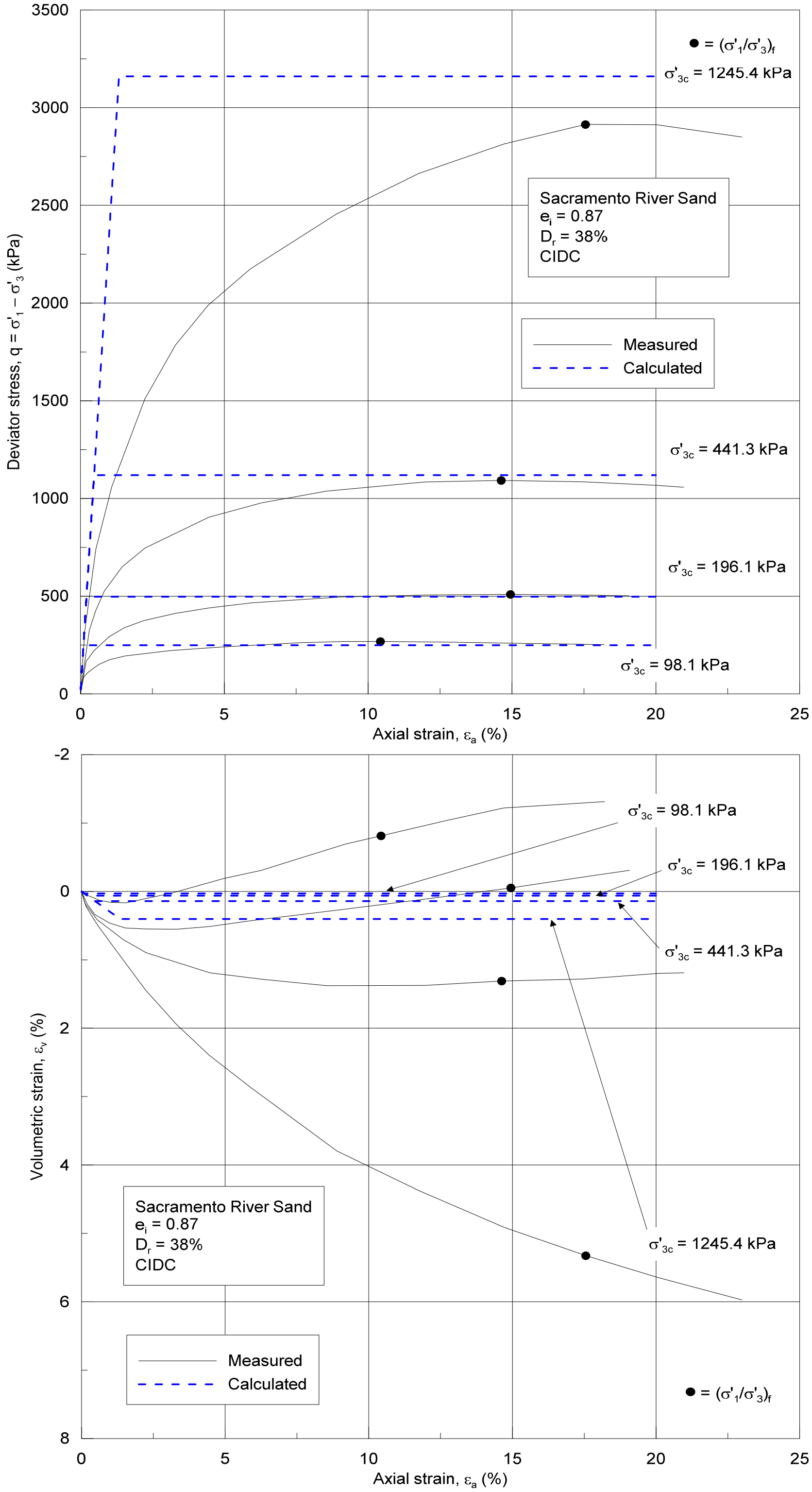


Figure 4.7 – CIDC stress-strain curves of Sacramento River Sand at D_r = 38%

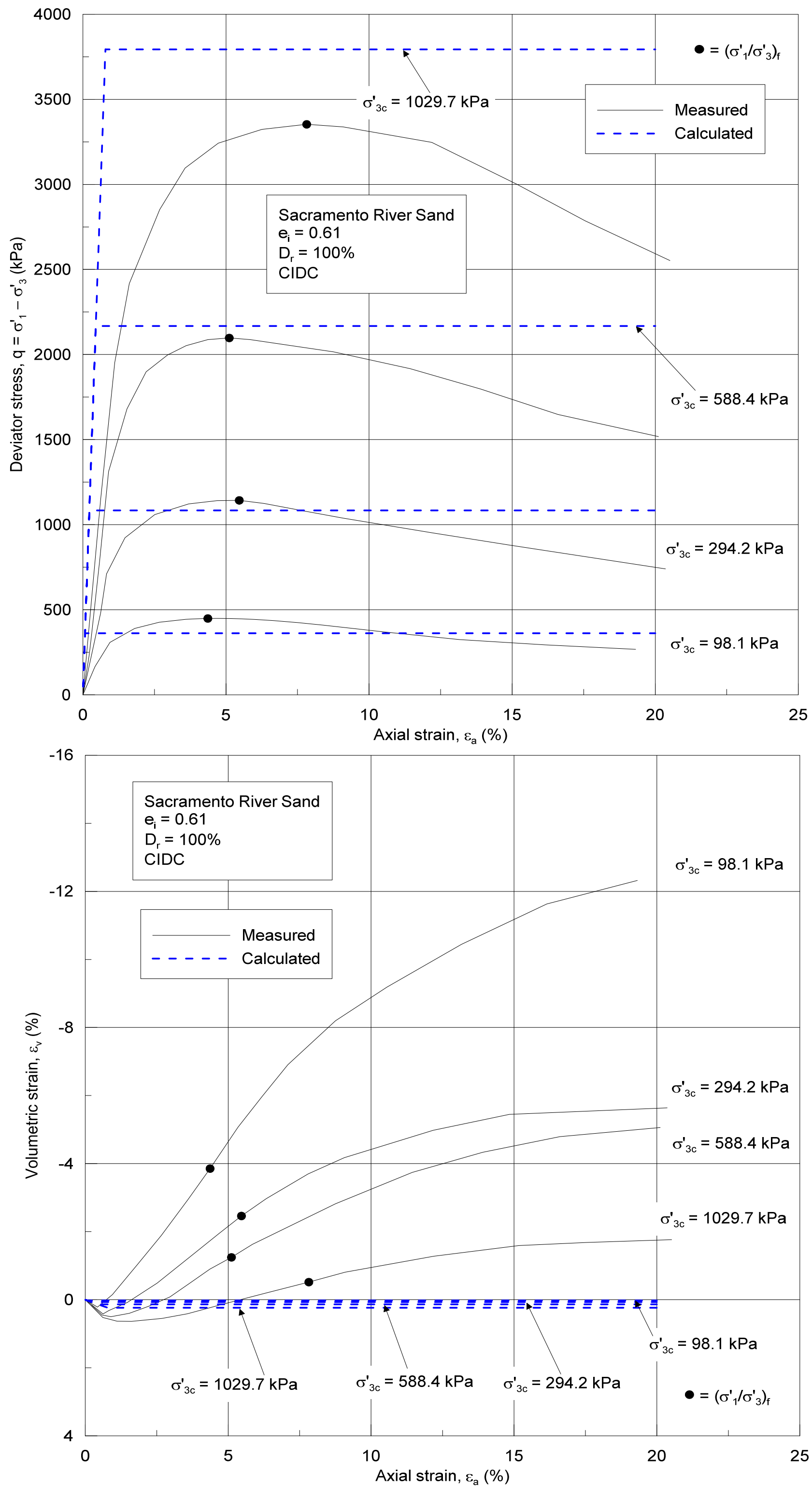


Figure 4.8 – CIDC stress-strain curves of Sacramento River Sand at D_r = 100%

Note that since a linear failure envelope was selected, the calculated shear strength can either over-predict or under-predict the measured behavior. When the friction angle was selected to fit the lower stress level, the FLAC Mohr-Coulomb model would over-predict the shear strength of the soil at higher stress levels and vice versa. This is evident in Figures 4.7 and 4.8.

4.3.2 Hostun Sand

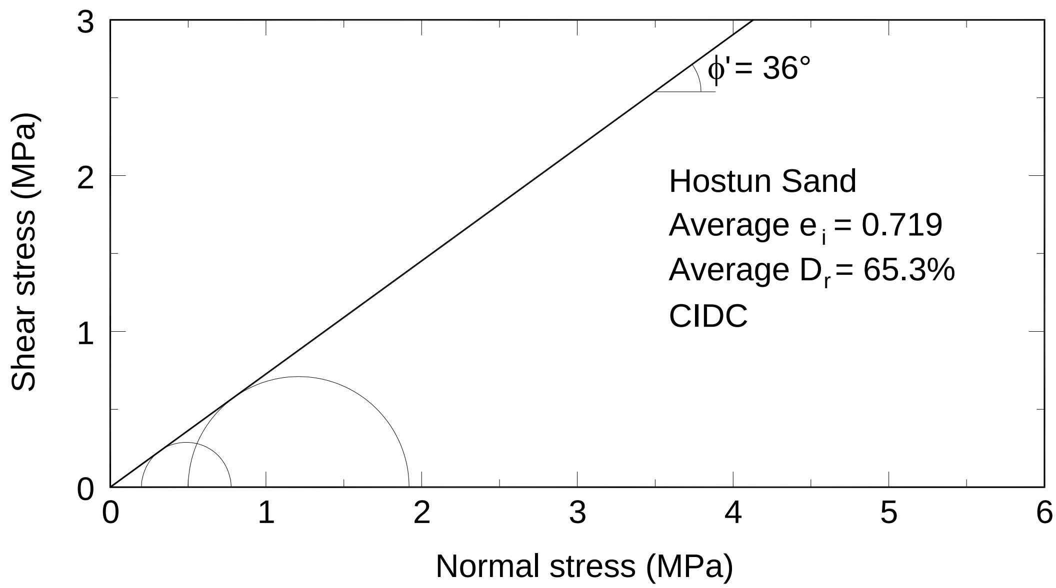
The four tests analyzed for Hostun Sand were CIDC, CIDE, CIDC constant p', and CIDE constant p'. Relative densities (D_r's) of 65.3% and 92.4% were evaluated for the CIDC and CIDE tests. D_r's of 63.7% and 92.7% were evaluated for CIDC constant p' and CIDE constant p' tests. The peak friction angles for D_r's of 65.3%, 92.4%, 63.7% and 92.7% were determined in Figures 4.9, 4.10, 4.11, and 4.12, respectively. It can be seen that the friction angles in triaxial compression are lower than friction angles in triaxial extension, which is in agreement with findings from other researchers. The influence of test boundary conditions is shown in Figure 4.13, in which the parameter b is termed intermediate effective principal stress factor and is defined as:

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} \tag{4.7}$$

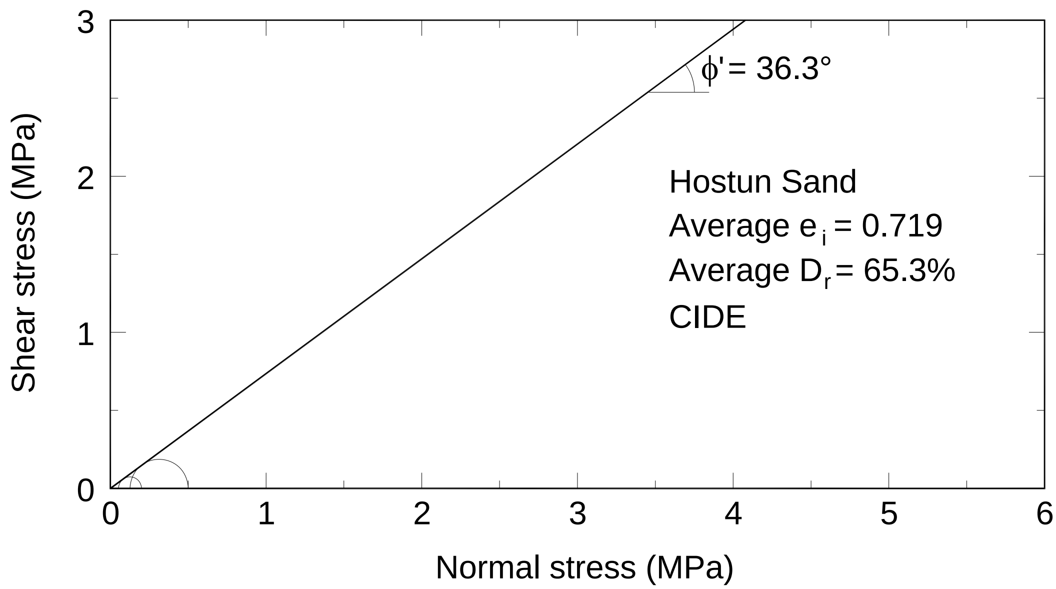
where σ'_1 and σ'_3 are major and minor principal stresses, respectively. Value of b varies from 0 to 1. In triaxial compression, $\sigma'_2 = \sigma'_3$, and b = 0. In triaxial extension, $\sigma'_2 = \sigma'_1$, hence b = 1. In most cases, ϕ'_p is lower in compression (b = 0) than in extension (b = 1), which suggests that the use of ϕ'_p from triaxial compression will be a conservative assumption. Although the friction angles from extension tests were identified, they are not used in this study; only the friction angles from the CIDC tests were specified in the study for the reasons given in Friction Angle and Dilation Angle section (Section 3.2.3.2).

The stress-strain curves for the CIDC and CIDE tests for D_r's of 65.3% and 92.4% are shown in Figures 4.14 and 4.15, respectively. The stress-strain curves for the CIDC and CIDE constant p' tests for D_r's of 63.7% and 92.7% are shown in Figures 4.16 and 4.17, respectively. The differences in measured and calculated stress-strain responses observed for Sacramento River Sand also apply to Hostun Sand. Additional differences include:

- Volumetric strain increases prior to failure and remains constant thereafter in the calculated CIDE response, whereas the measured behavior shows initial contraction and followed by dilation.
- The calculated CIDC and CIDE constant p' responses show zero volumetric strain during shearing, but the measured behaviors show differently.

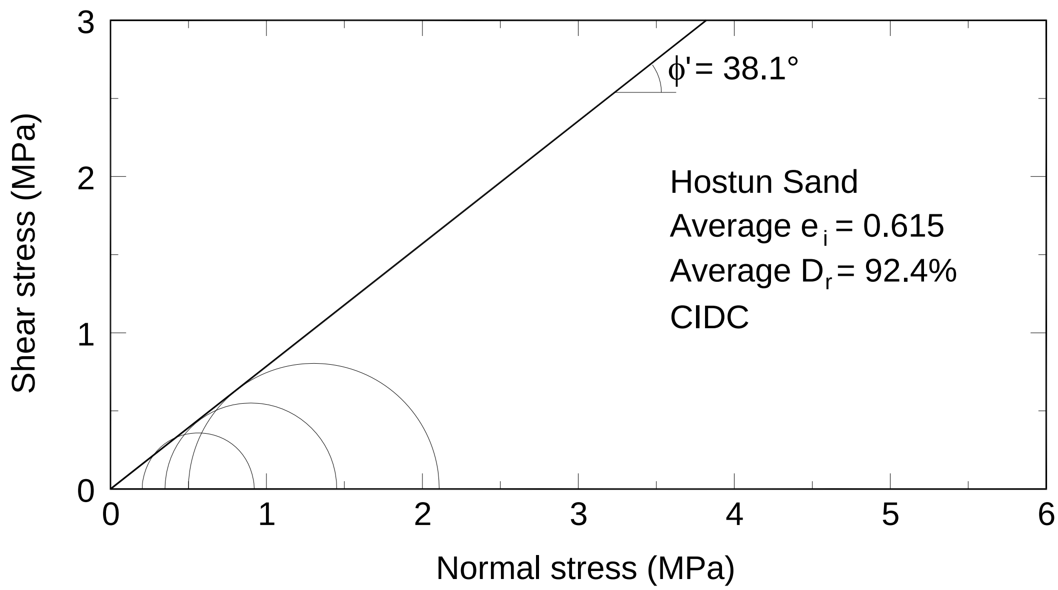


(a)



(b)

Figure 4.9 – Mohr circles at failure of Hostun Sand at D_r = 65.3% for (a) CIDC and (b) CIDE tests



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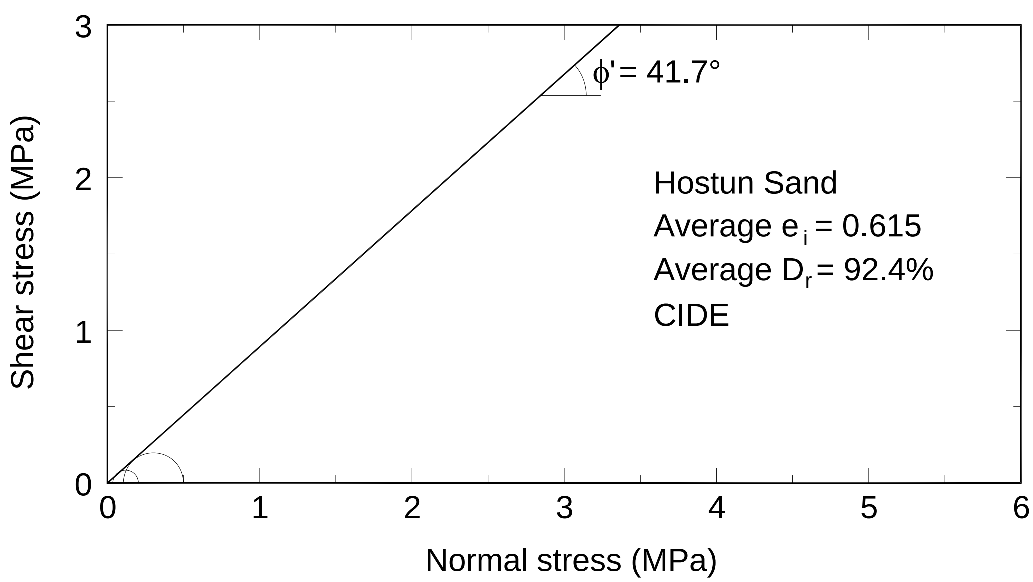
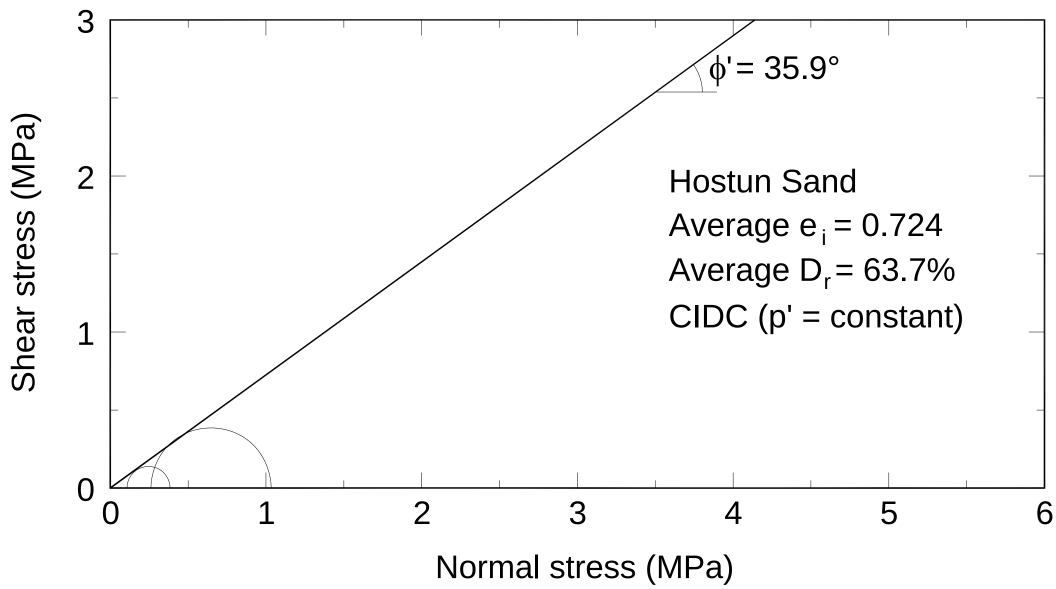
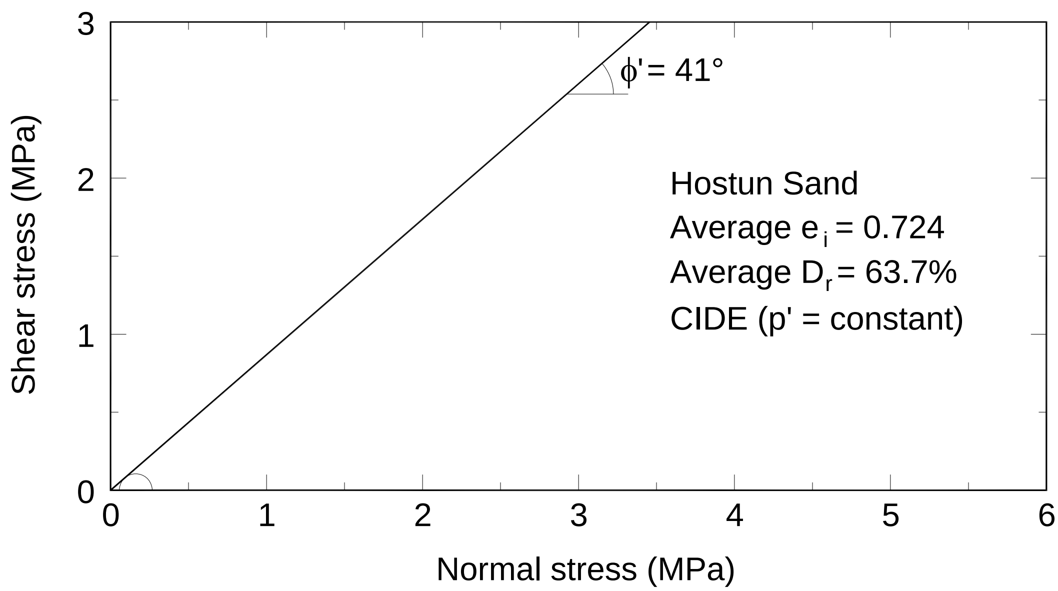


(b)

Figure 4.10 – Mohr circles at failure of Hostun Sand at D_r = 92.4% for (a) CIDC and (b) CIDE tests

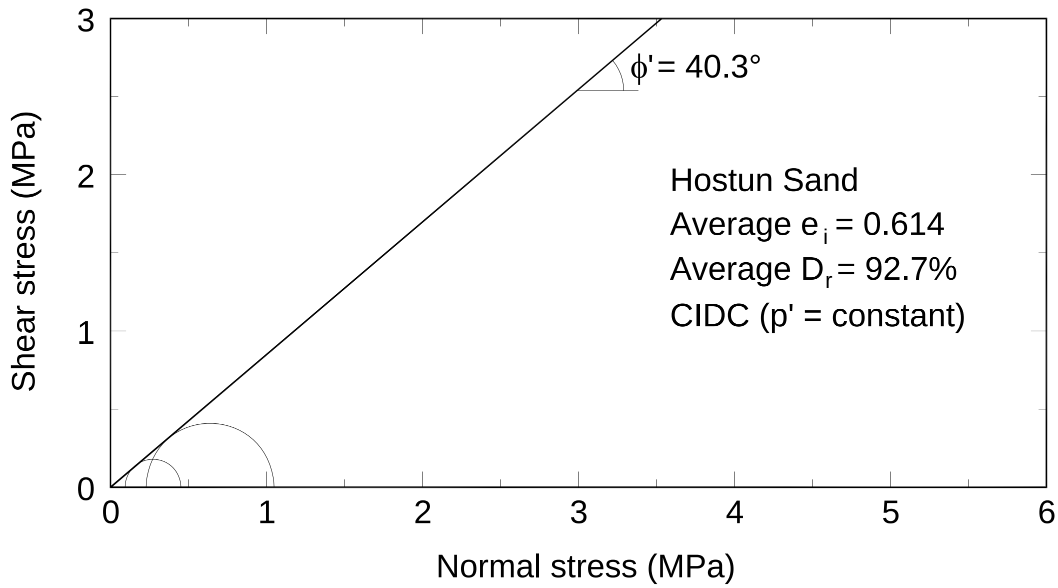


(a)

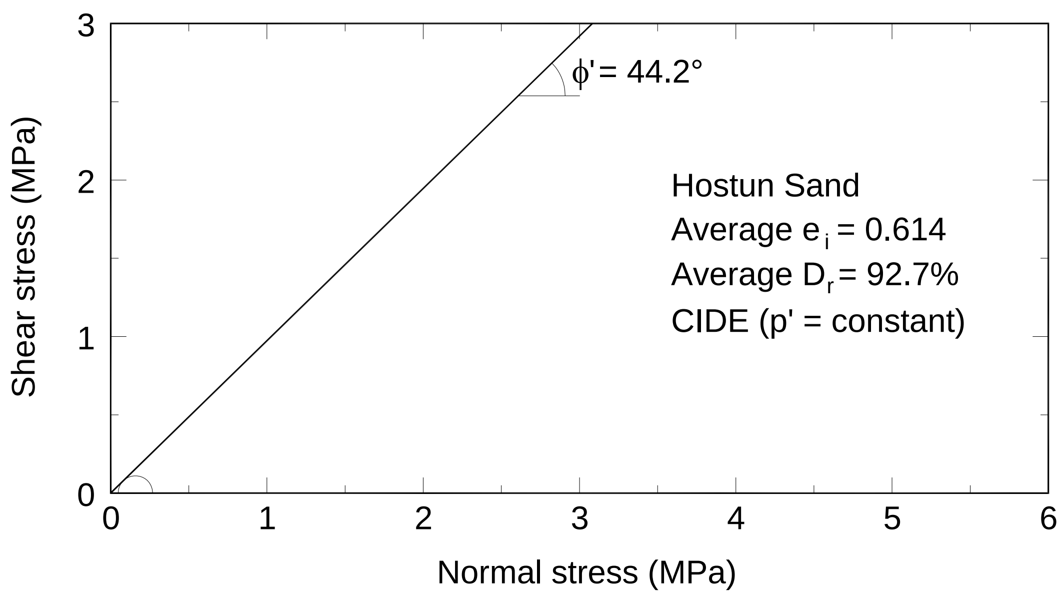


(b)

Figure 4.11 – Mohr circles at failure of Hostun Sand at D_r = 63.7% for (a) CIDC constant p' and (b) CIDE constant p' tests



(a)



(b)

Figure 4.12 – Mohr circles at failure of Hostun Sand at D_r = 92.7% for (a) CIDC constant p' and (b) CIDE constant p' tests

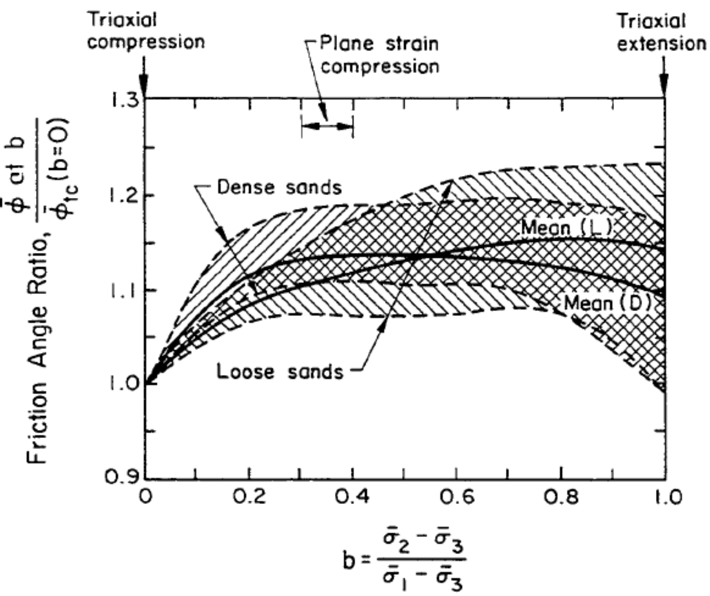


Figure 4.13 – Influence of intermediate principal stress on friction angle (after Kulhawy and Mayne [15])

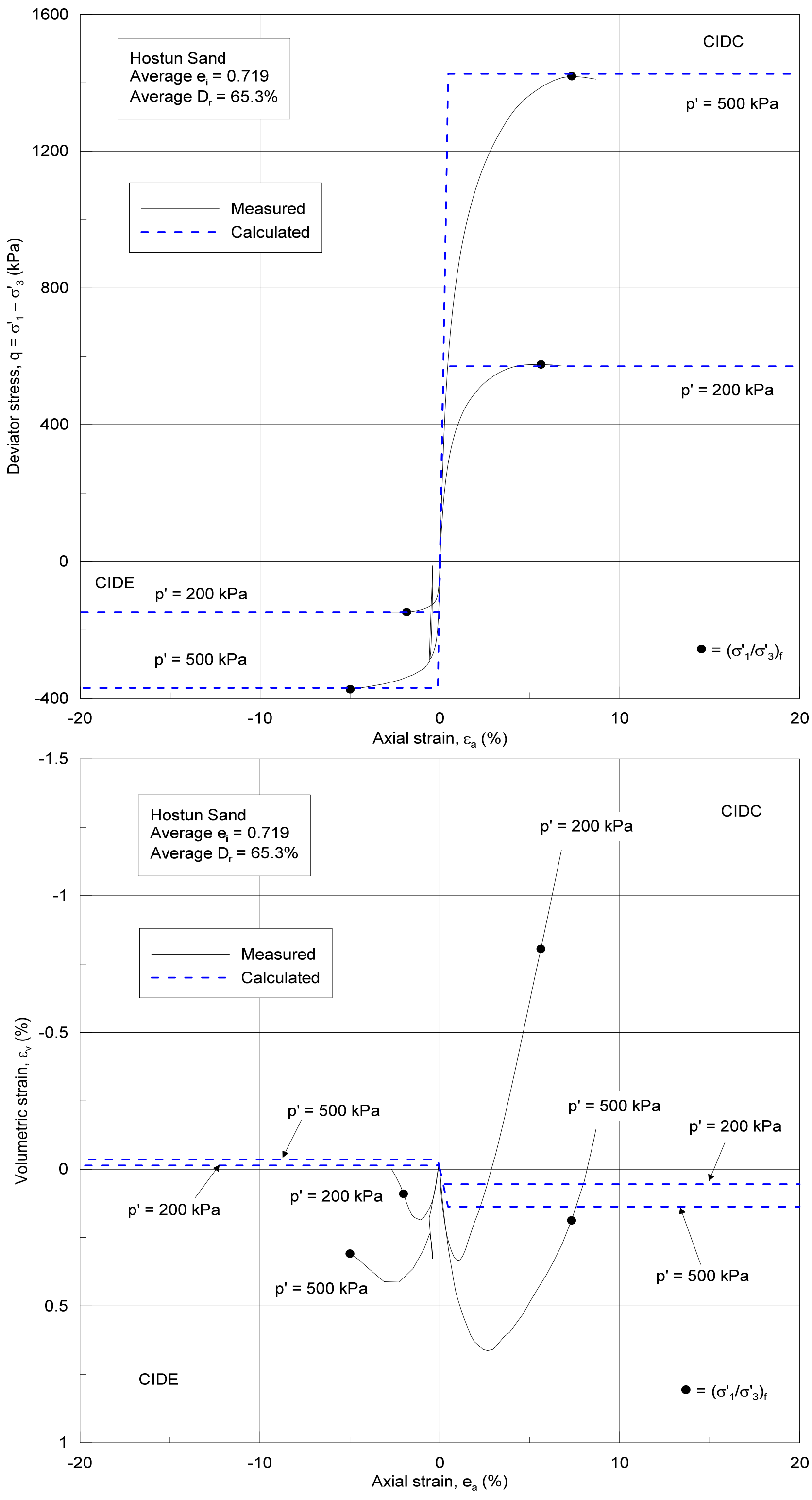


Figure 4.14 – CIDC and CIDE stress-strain curves of Hostun Sand at D_r = 65.3%

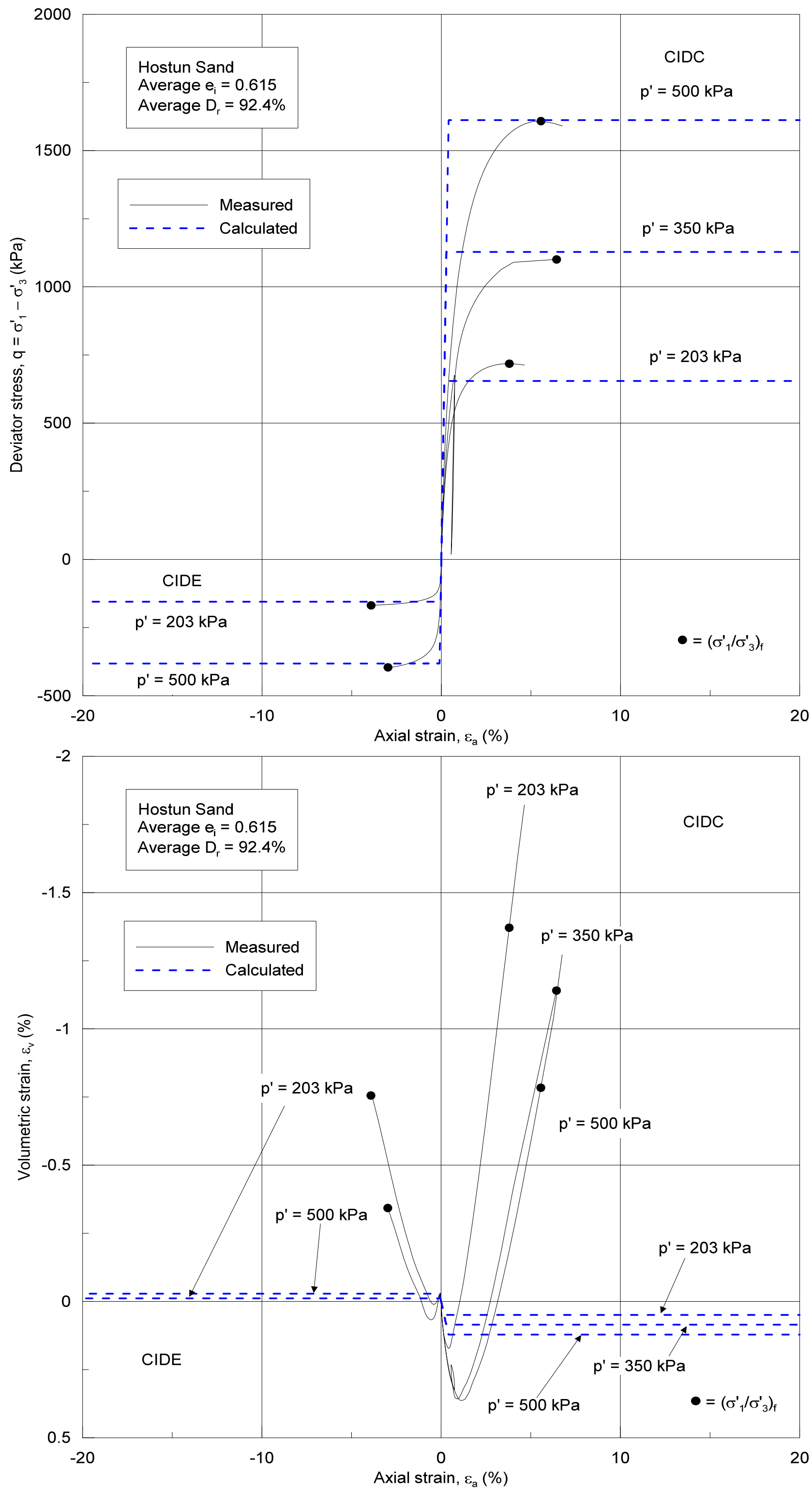


Figure 4.15 – CIDC and CIDE stress-strain curves of Hostun Sand at D_r = 92.4%

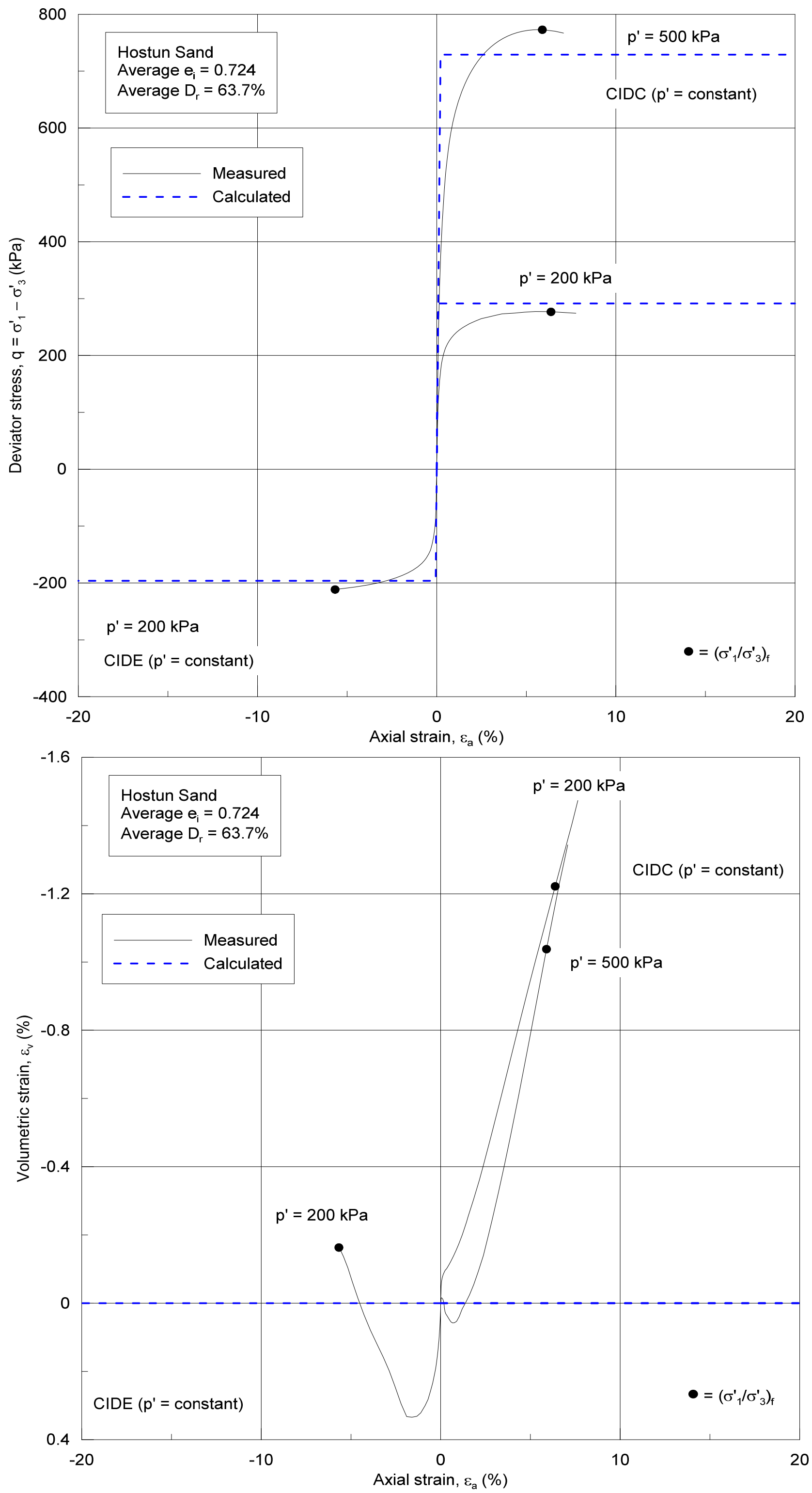


Figure 4.16 – CIDC and CIDE at constant p' stress-strain curves of Hostun Sand at D_r = 63.7%

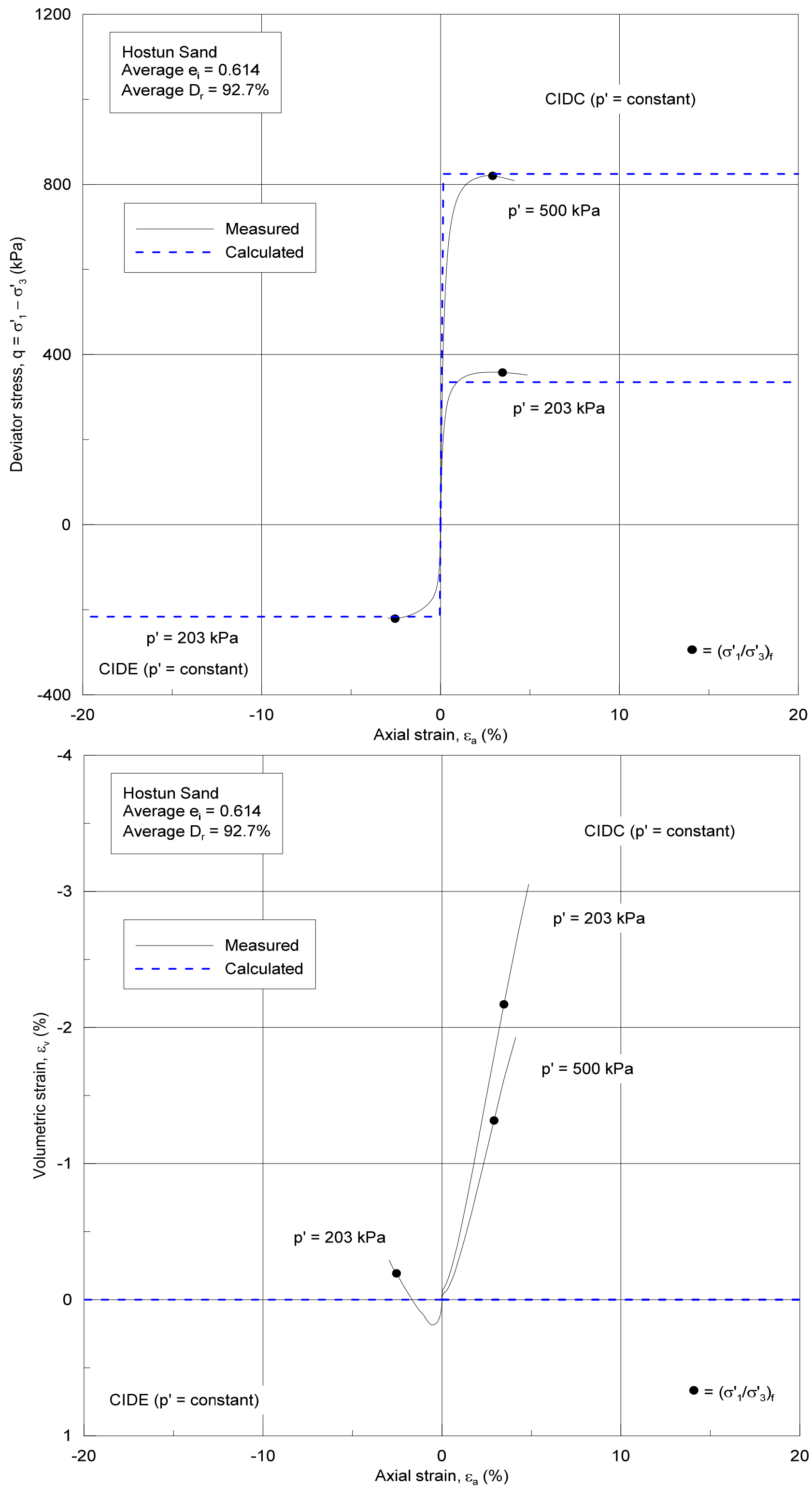


Figure 4.17 – CIDC and CIDE at constant p' stress-strain curves of Hostun Sand at D_r = 92.7%

4.3.3 Nevada Sand

The four tests analyzed for Nevada Sand were oedometer, CIDC constant p', CIDE constant p', and direct simple shear (DSS) tests. The two relative densities considered were 40% and 60%. The peak friction angles from Mohr circles at failure for the two relative densities are shown in Figures 4.18 and 4.19. The peak friction angles from the CIDC constant p' tests were specified in the numerical analyses. Note that, although not used, a low friction angle is observed in the CIDE constant p' test at D_r of 60%, which may be an anomaly.

The stress-strain curves of oedometer tests are compared in Figure 4.20. It is observed that the calculated stress-strain curves are linear, whereas the measured stress-strain curves are nonlinear. A much stiffer response is observed in the calculated curves than the measured curves. The slope of the calculated curve is the constrained modulus. The hysteretic loops are not observed in the calculated response during loading and unloading cycles.

The stress-strain curves for CIDC and CIDE at constant p' tests for D_r's of 40% and 60% are shown in Figures 4.21 and 4.22, respectively. The differences in measured and calculated stress-strain responses observed for Sacramento River Sand and Hostun Sand also apply to Nevada Sand. Note that, from the measured behavior, the deviator stress induced volumetric strains appear to be independent of mean effective stress p', and the same is observed in Hostun Sand.

The DSS tests analyzed are constant-height drained tests. Initially, the soil sample was consolidated under a predetermined vertical effective stress. During the test, vertical movement of the sample was restrained, while the sample was subjected to shear stress under drained conditions. The tests were conducted under plane strain condition. The stress-strain relationships of DSS tests subjected to vertical consolidation stress of 80 kPa are compared in Figures 4.23 and 4.24 for D_r's of 40% and 60%, respectively. In the post processing of FLAC output, the full strain rate tensor (i.e., FISH function fsi(i,j,4), index 4 = xy component) was used to calculate the engineering shear strain as:

$$\gamma = 2\ \varepsilon_{xy} \tag{4.8}$$

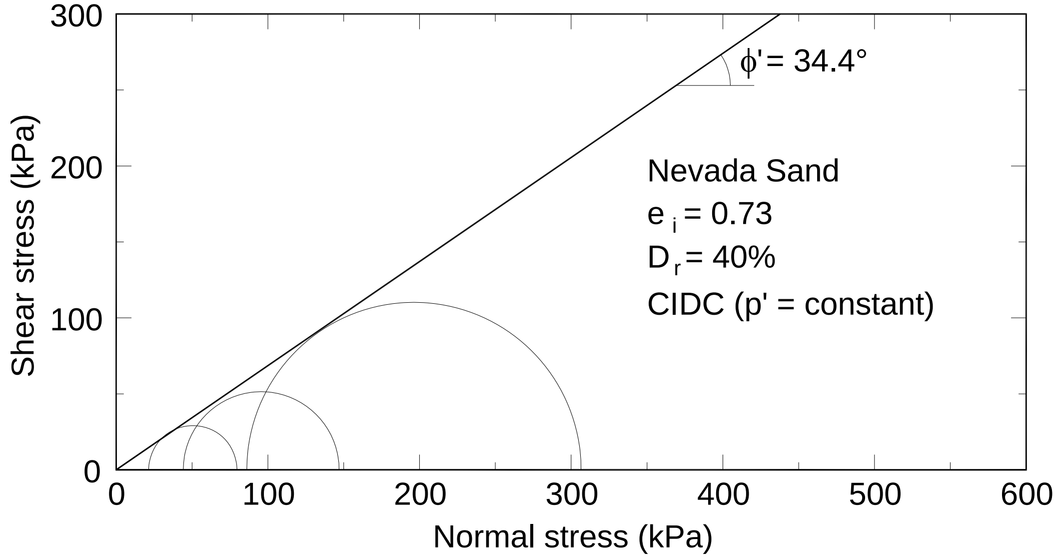
where γ = engineering shear strain and ε_{xy} = pure shear strain.

The differences between the measured and calculated stress-strain responses include:

- Nonlinear soil behavior is not captured by FLAC Mohr-Coulomb model.
- The elastic-perfectly plastic stress-strain response is implemented in the FLAC Mohr-Coulomb model.
- The FLAC Mohr-Coulomb model significantly under-predicts shear stress at large strains.

- The vertical effective stress in laboratory DSS test varies with increasing shear stress and shear strain. The vertical stress decreases initially and increases when shear strain reaches about 2%. This behavior is not captured by the FLAC Mohr-Coulomb model. The calculated vertical effective stress decreased until failure occurred in shear stress, and the calculated vertical effective stress at failure was higher than the measured behavior.

It should be noted that although the laboratory DSS tests were intended as constant-height (or constant volume) drained tests, increases in volumetric strain during shearing were measured. The laboratory measurements suggest that the constant-height was not maintained during the test, and the increases in volumetric strain indicate that dilation had occurred during shearing. The dilation had caused increases in shear stress and vertical stress as shown in Figures 4.23 and 4.24. The discrepancy between the measured and FLAC calculated stress-strain responses is thought to be attributed mainly to the differences in boundary conditions.



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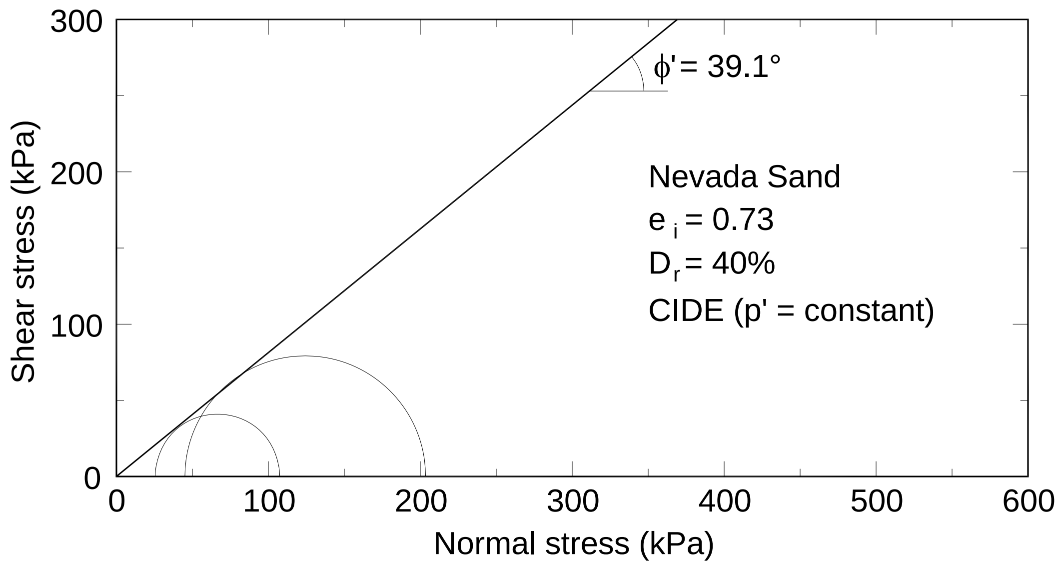
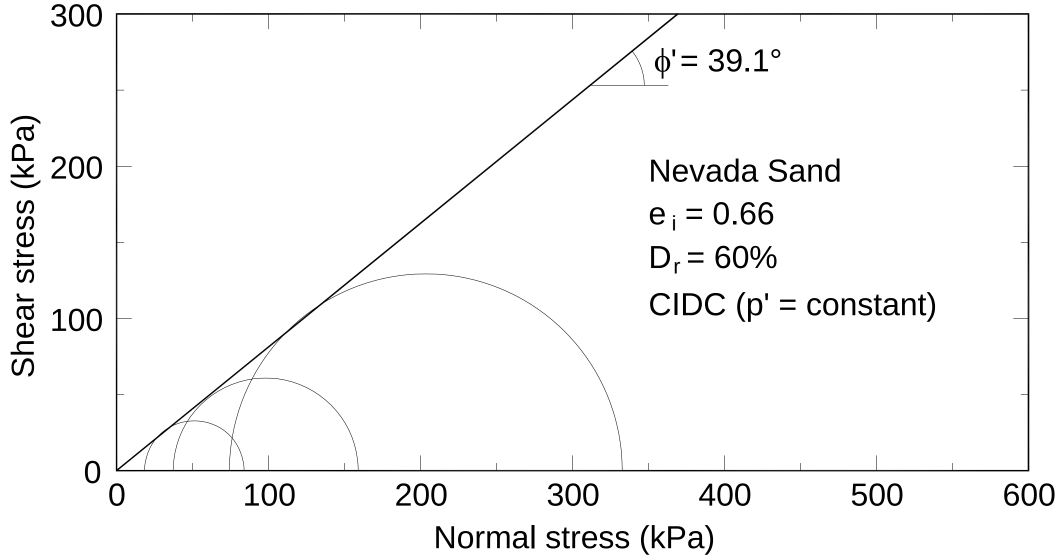
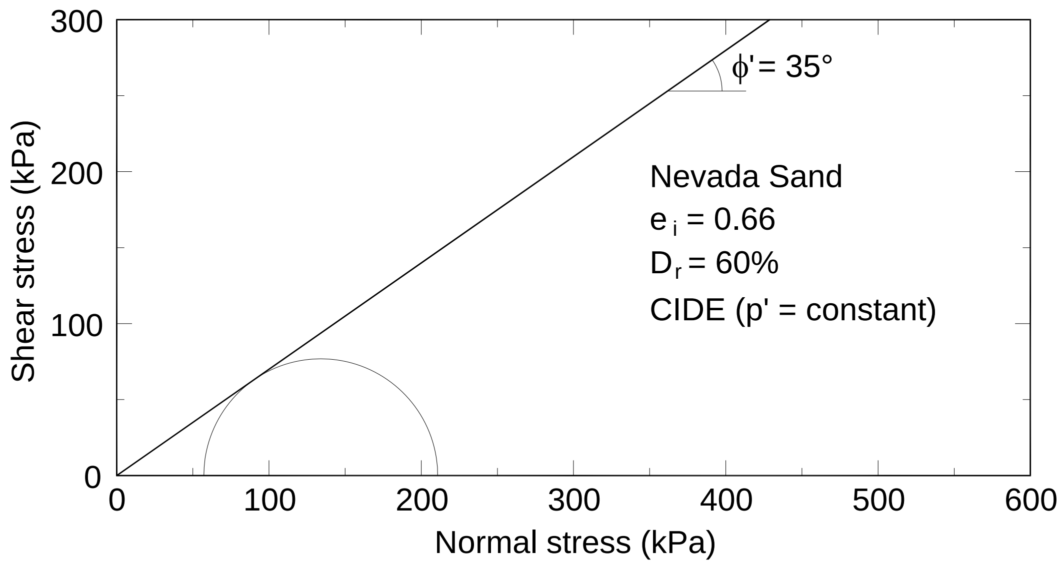


(b)

Figure 4.18 – Mohr circles at failure of Nevada Sand at D_r = 40% for (a) CIDC constant p' and (b) CIDE constant p' tests

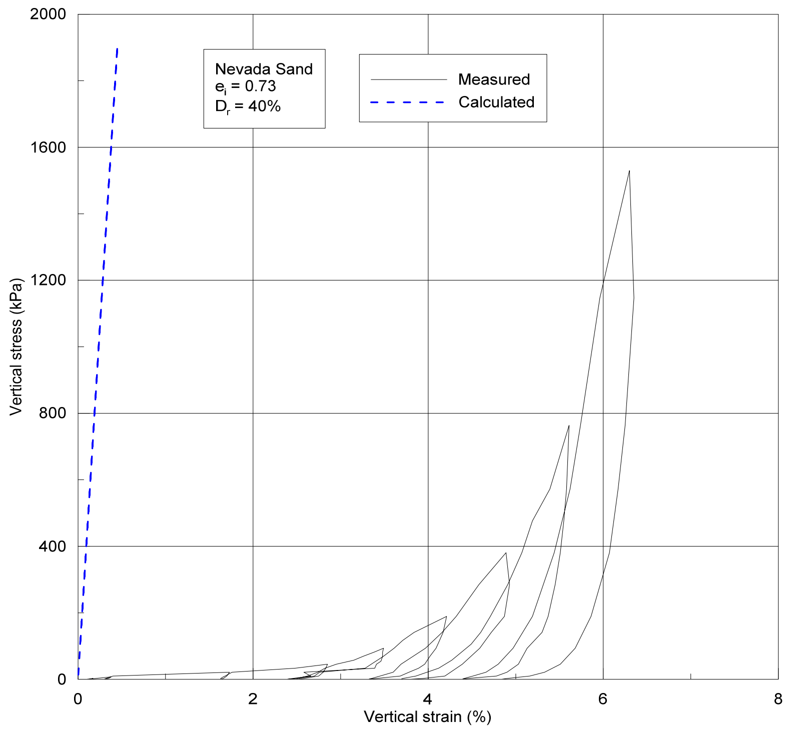


(a)

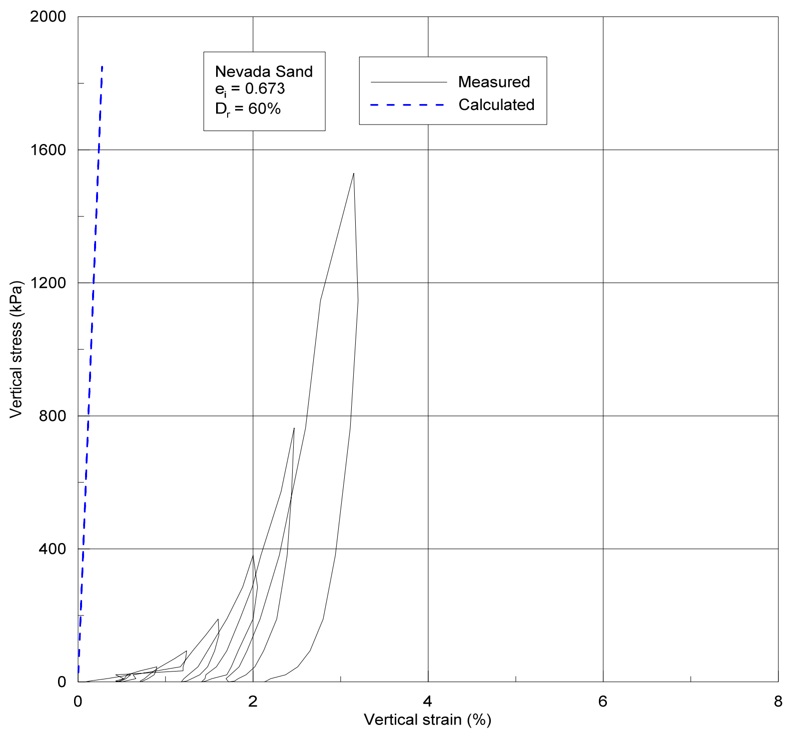


(b)

Figure 4.19 – Mohr circles at failure of Nevada Sand at D_r = 40% for (a) CIDC constant p' and (b) CIDE constant p' tests



(a)



(b)

Figure 4.20 – Oedometer stress-strain curves of Nevada Sand at (a) D_r = 40% and (b) D_r = 60%

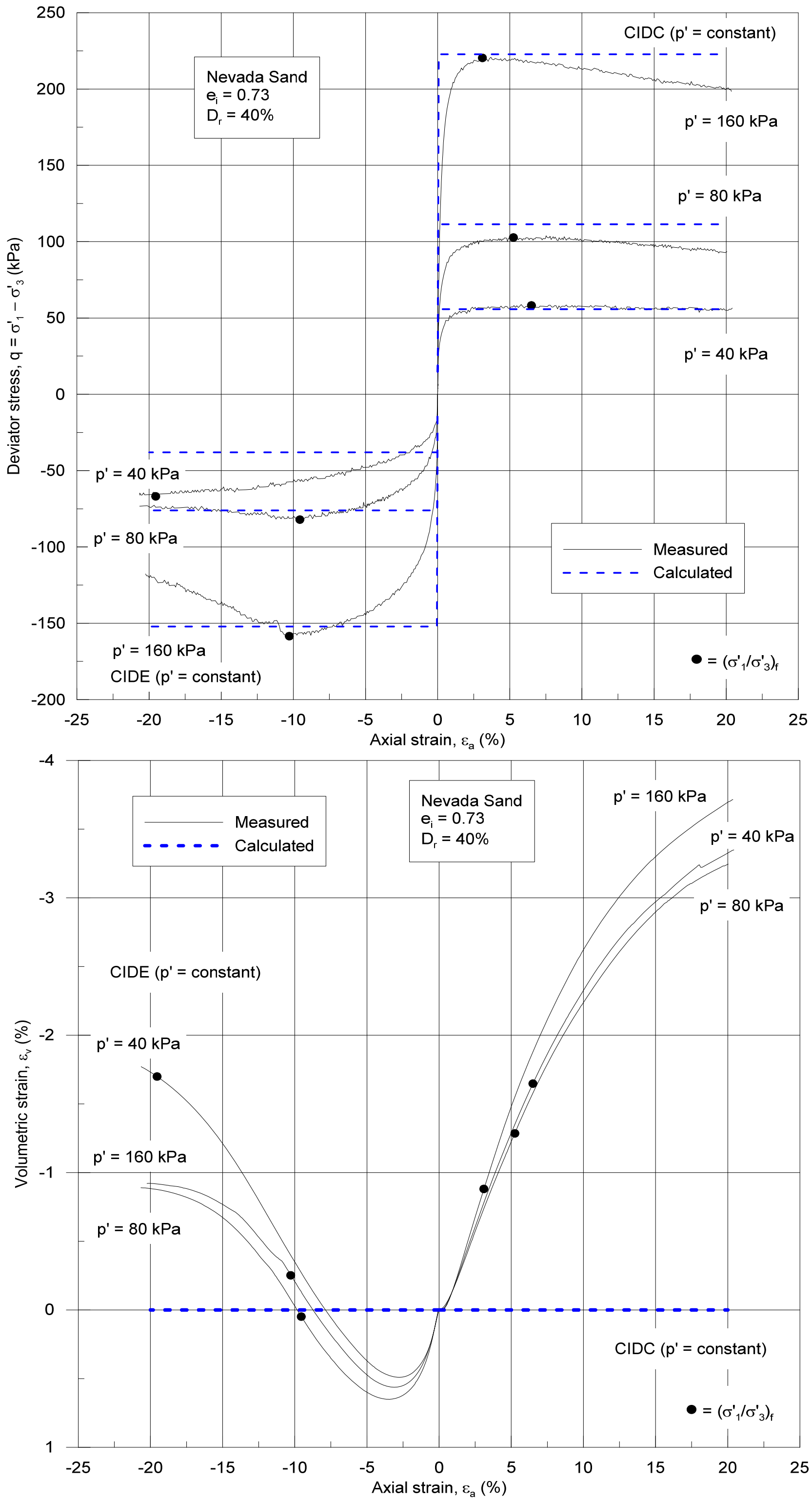


Figure 4.21 – CIDC and CIDE at constant p' stress-strain curves of Nevada Sand at D_r = 40%

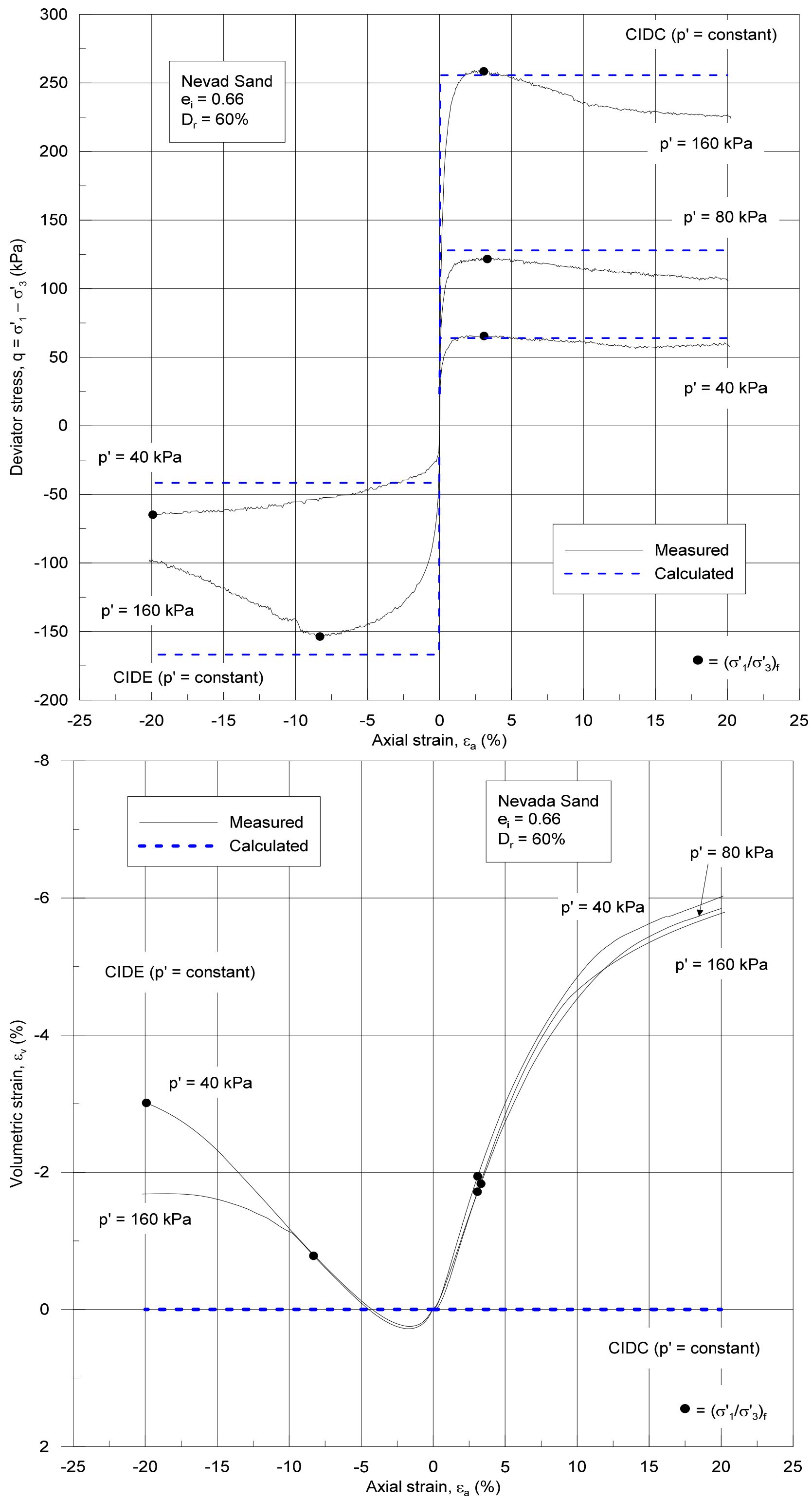


Figure 4.22 – CIDC and CIDE at constant p' stress-strain curves of Nevada Sand at D_r = 60%

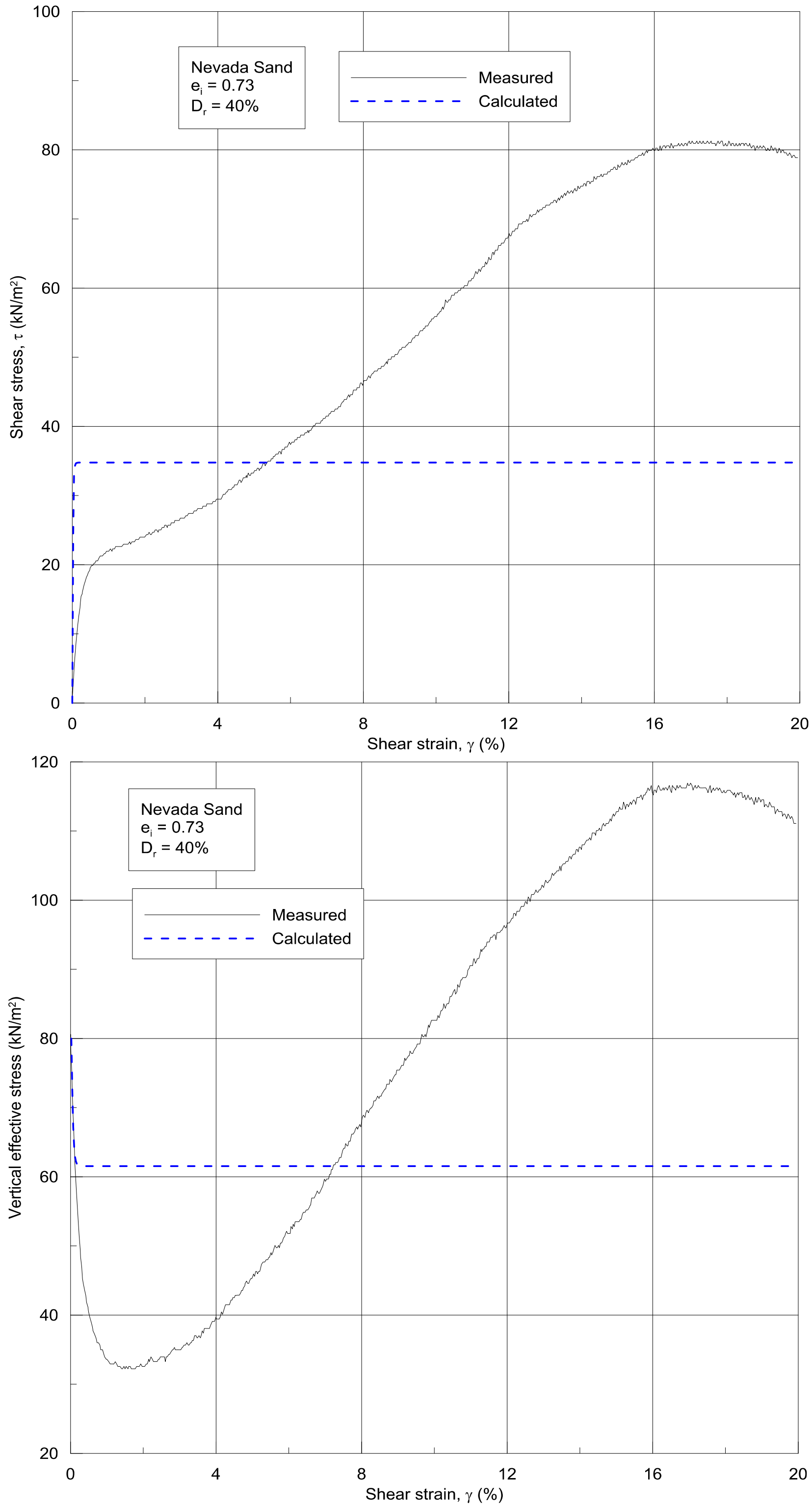


Figure 4.23 – DSS constant-height stress-strain curves of Nevada Sand at D_r = 40%

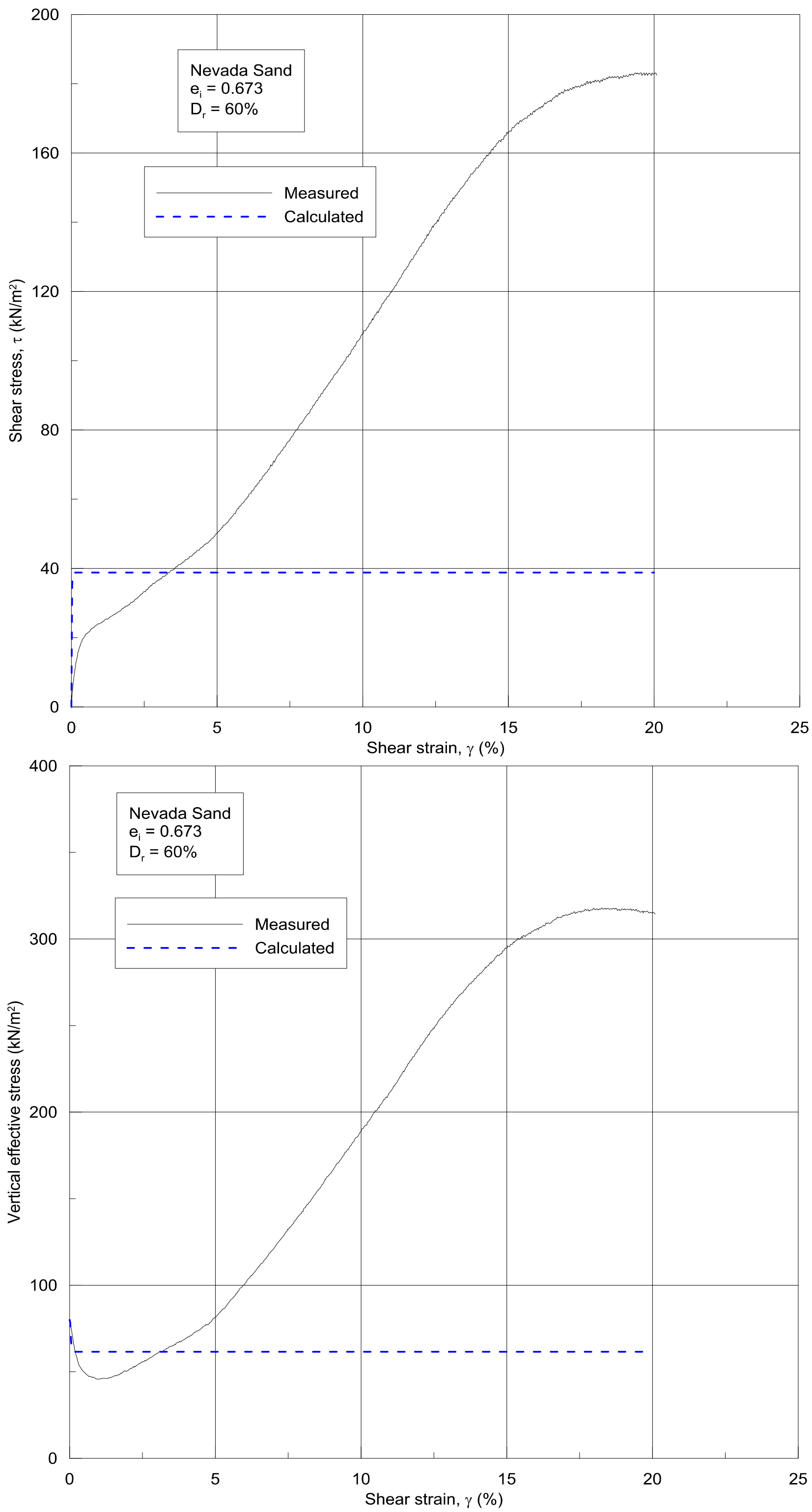


Figure 4.24 – DSS constant-height stress-strain curves of Nevada Sand at D_r = 60%

5. PARAMETRIC STUDY

The two parameters evaluated are dilation angle and hysteretic damping. Current Reclamation practice is to assume dilation angle to be zero, since dilation angle is generally not readily available. As shown later in the section, dilation angle depends on the critical state friction angle. Estimating model parameters other than cohesion and peak friction angle introduces more uncertainties and unknowns and can be cost prohibitive in the routine material characterization process. Hysteretic damping has been assigned for the foundation bedrock material with the recognition that it is applicable for material experiencing small strains during dynamic loading. This section evaluates the effects of dilation angle and hysteretic damping on the Mohr-Coulomb model. The FLAC input files of the parametric study are included in the Appendix.

5.1 Dilation Angle

The term dilatancy is used to describe the tendency of soil to expand or contract in volume upon shearing. It has been recognized that dense sand expands and loose sand contracts when sheared. According to Vaid and Sasitharan [24], dilation angle ψ can be defined as:

$$\psi = \sin^{-1}\left[\frac{-d\varepsilon_v}{d\gamma}\right] = \sin^{-1}\left[\frac{-d\varepsilon_v}{d\varepsilon_1 - d\varepsilon_3}\right] \qquad (5.1)$$

where $d\varepsilon_v$ = volumetric strain increment, $d\gamma$ = engineering shear strain increment, $d\varepsilon_1$ = major principal strain increment, and $d\varepsilon_3$ = minor principal strain increment. A positive ψ indicates an increase in volume or dilation, while a negative ψ indicates a reduction in volume or compression. The negative sign in Equation 5.1 is needed since a negative change in volume (i.e., expansion) corresponds to a positive rate of dilation. In triaxial compression,

$$d\varepsilon_1 = d\varepsilon_a \qquad (5.2)$$

and

$$d\varepsilon_3 = d\varepsilon_r = \frac{d\varepsilon_v - d\varepsilon_a}{2} \qquad (5.3)$$

where $d\varepsilon_a$ = axial strain increment and $d\varepsilon_r$ = radial strain increment. Substituting Equations 5.2 and 5.3 in Equation 5.1 gives:

$$\psi = \sin^{-1}\left[\frac{2}{1-3/(d\varepsilon_v / d\varepsilon_a)}\right] \qquad (5.4)$$

The contribution of dilation angle to shear resistance of a soil along a failure surface can be represented in the sawtooth model shown in Figure 5.1. The angle of shearing resistance $\phi'_{current}$ mobilized at some stage during the shearing process can be determined as:

$$\phi'_{current} = \phi'_{cv} + \psi_{current} \tag{5.5}$$

where ϕ'_{cv} = critical state (or constant void ratio) friction angle and $\psi_{current}$ = current dilation angle. The critical state occurs when the void ratio become constant as shearing continues, and the critical void ratio is the void ratio when volumetric strain is zero (i.e., $d\varepsilon_v/d\gamma = 0$). Consequently, the peak angle of shearing resistance ϕ'_p is accompanied by the peak dilation angle ψ_p as:

$$\phi'_p = \phi'_{cv} + \psi_p \tag{5.6}$$

However, as shown by the experimental data, Equation 5.6 overestimates the effect of dilation on peak strength. For the plane strain condition, Bolton [25] showed that the contribution of dilation to peak strength can be approximated by the empirical equation:

$$\phi'_p = \phi'_{cv} + 0.8\psi_p \tag{5.7}$$

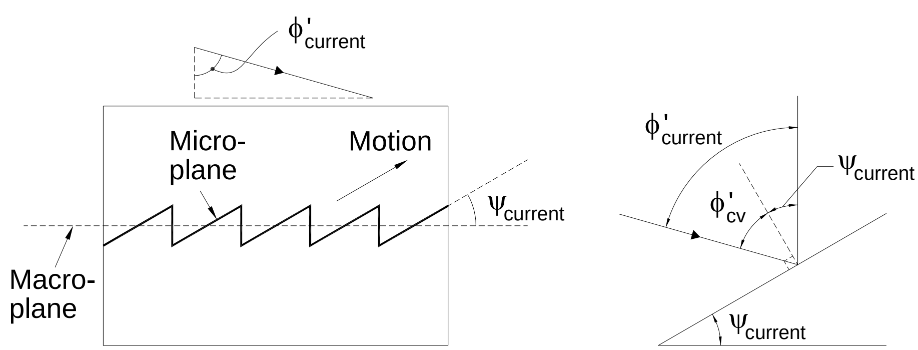


Figure 5.1 – The sawtooth model for dilatancy (after Bolton [25])

5.1.1 Determination of Dilation Angle

As shown by Bishop [26] and Vaid and Sasitharan [24], there exists a unique relationship between the peak friction angle ϕ'_p and the peak dilation angle ψ'_p in conventional triaxial compression tests regardless of stress path, confining stress at failure, relative density, and the mode of loading (compression or extension). The derivation developed by Vaid and Sasitharan was used to estimate ψ'_p for Sacramento River Sand. The procedure is described as follows.

For each CIDC test, the maximum principal stress ratio, $(\sigma'_1/\sigma'_3)_f$, as the failure condition was determined, and the corresponding σ'_{1f} and σ'_{3f} were used to find ϕ'_p as:

$$\phi'_p = \sin^{-1}\left[\frac{\sigma'_{1f} - \sigma'_{3f}}{\sigma'_{1f} + \sigma'_{3f}}\right] \tag{5.8}$$

The change in volumetric strain $d\varepsilon_v$ and the change in axial strain $d\varepsilon_a$ at the failure condition $(\sigma'_1/\sigma'_3)_f$ were also determined to find ψ_p as:

$$\psi_p = \sin^{-1}\left[\frac{2}{1 - 3/(d\varepsilon_v / d\varepsilon_a)}\right] \tag{5.4bis}$$

Table 5.1 summarizes the CIDC experimental data needed to find ϕ'_p and ψ_p for Sacramento River Sand. Figure 5.2 shows the variation of ϕ'_p with ψ_p. Extrapolation of ϕ'_p and ψ_p relation with $\psi_p = 0°$ should yield the critical state friction angle ϕ'_{cv} of Sacramento River Sand. The linear regression equation in Figure 5.2 was found to be:

$$\phi'_p = \phi'_{cv} + 0.31\psi_p \tag{5.9}$$

with $\phi'_{cv} = 34.4°$. For this parametric study, the peak friction angle of Sacramento River Sand at $D_r = 100\%$ is 40.4° (i.e., $\phi'_p = 40.4°$). Utilizing Equation 5.9, ψ_p was found to be 19.4° and was specified as the dilation angle for the Mohr-Coulomb model.

Table 5.1 – CIDC data of Sacramento River Sand to find ϕ'_p and ψ_p

D_r (%)	σ'_{3c} (kPa)	ϕ'_p (deg)	$-(d\varepsilon_v/d\varepsilon_a)_p$	ψ_p (deg)
38	98.1	35.29	0.098	3.64
38	196.1	34.41	0.059	2.21
38	441.3	33.58	0.017	0.65
38	1245.4	32.63	-0.137	-5.48
100	98.1	44.11	1.211	35.12
100	294.2	41.32	0.622	20.09
100	588.4	39.84	0.492	16.36
100	1029.7	38.28	0.221	7.88

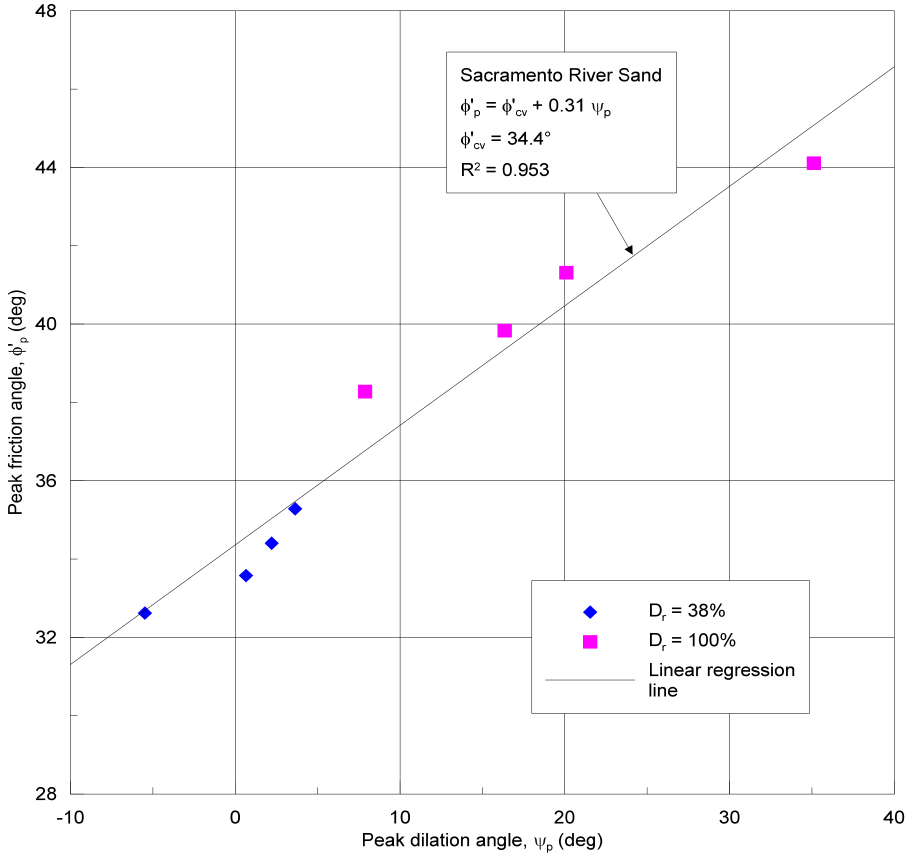


Figure 5.2 – Variation of ϕ'_p with ψ_p from CIDC tests of Sacramento River Sand

The effects of dilation angle on performances of geotechnical structures (e.g., soil slope, footing, tunnel lining, and pile) have been discussed by Houlsby [27]. It can be seen that the effect of dilation angle is more pronounced when the soil is kinematically constrained. For instance, bearing capacity of a pile increases dramatically with increasing dilation angle, since a pile is highly confined. On the other hand, the safety factor of a slope stability problem is not influenced by the dilation angle, since soil slope is relatively unconstrained. Based on the insignificant influence of dilation angle on soil slope stability, performance of an embankment dam is not expected to be significantly affected by the dilation angle; however, this claim has not been proven.

5.1.2 Effects of Dilation Angle

The CIDC tests of Sacramento River Sand with D_r of 100% were simulated using the FLAC Mohr-Coulomb model with a dilation angle (ψ_p = 19.4°) specified. The stress-strain curves of the CIDC tests are compared in Figure 5.3. No difference is observed in the calculated deviator stress between zero dilation angle and ψ_p = 19.4°. However, a volumetric expansion is observed when a dilation angle was specified, where the sand dilates when the maximum shear strength is reached. The differences between measured and calculated volumetric strain include:

- In the calculated response with dilation angle specified, the rate of volumetric expansion is constant after failure.
- FLAC Mohr-Coulomb model overestimates the volumetric expansion, especially for the higher confining stresses (e.g., $\sigma'_{3c} > 98.1$ kPa).
- Strain softening is not captured by the Mohr-Coulomb model.
- Constant volumetric strain is attained at large axial strain in the measured data (e.g., $\varepsilon_a > 15\%$), whereas the calculated volumetric strain increases linearly with increasing axial strain.

5.2 Hysteretic Damping

FLAC hysteretic damping implements the concept of the equivalent-linear model approach. The equivalent-linear method utilizing degradation curves (i.e., secant shear modulus and damping ratio curves) has been used to calculate wave propagation and response spectra in soil and rock when subjected to seismic excitation (e.g., SHAKE analysis). FLAC hysteretic damping option incorporates the strain-dependent secant modulus and damping ratio functions to better simulate energy dissipation and to improve nonlinear stress-strain behavior of a material model that does not produce hysteretic damping by itself such as the Mohr-Coulomb model. By activating the hysteretic damping option, the shear modulus is gradually reduced as strain increases according to the assigned modulus reduction curve. The hysteretic damping is switched off for each FLAC zone when plastic flow occurs.

FLAC hysteretic damping can be simulated by various built-in modulus functions. The sigmoidal model (i.e., sig3 model) was chosen for the Sacramento River Sand in the parametric study. The three input parameters required by the sig3 model are: a, b, and x_o. Numerical fits to the modulus reduction and damping ratio curves of sand published by Seed and Idriss [28] have been reported in the FLAC Dynamic Analysis manual [29], and the published parameters are a = 1.014, b = –0.4792, and x_o = –1.249. These best fit parameters were adopted for this study. The comparisons between the built-in sig3 functions and the literature curves are shown in Figure 5.4.

As stated in the FLAC Dynamic Analysis manual [29], the hysteretic damping formulation should not be used as a primary way to simulate yielding; the hysteretic damping option is only intended to provide damping for material models lacking intrinsic damping when not yielding. Furthermore, hysteretic damping will cause excessive reduction in shear modulus if the strain is large.

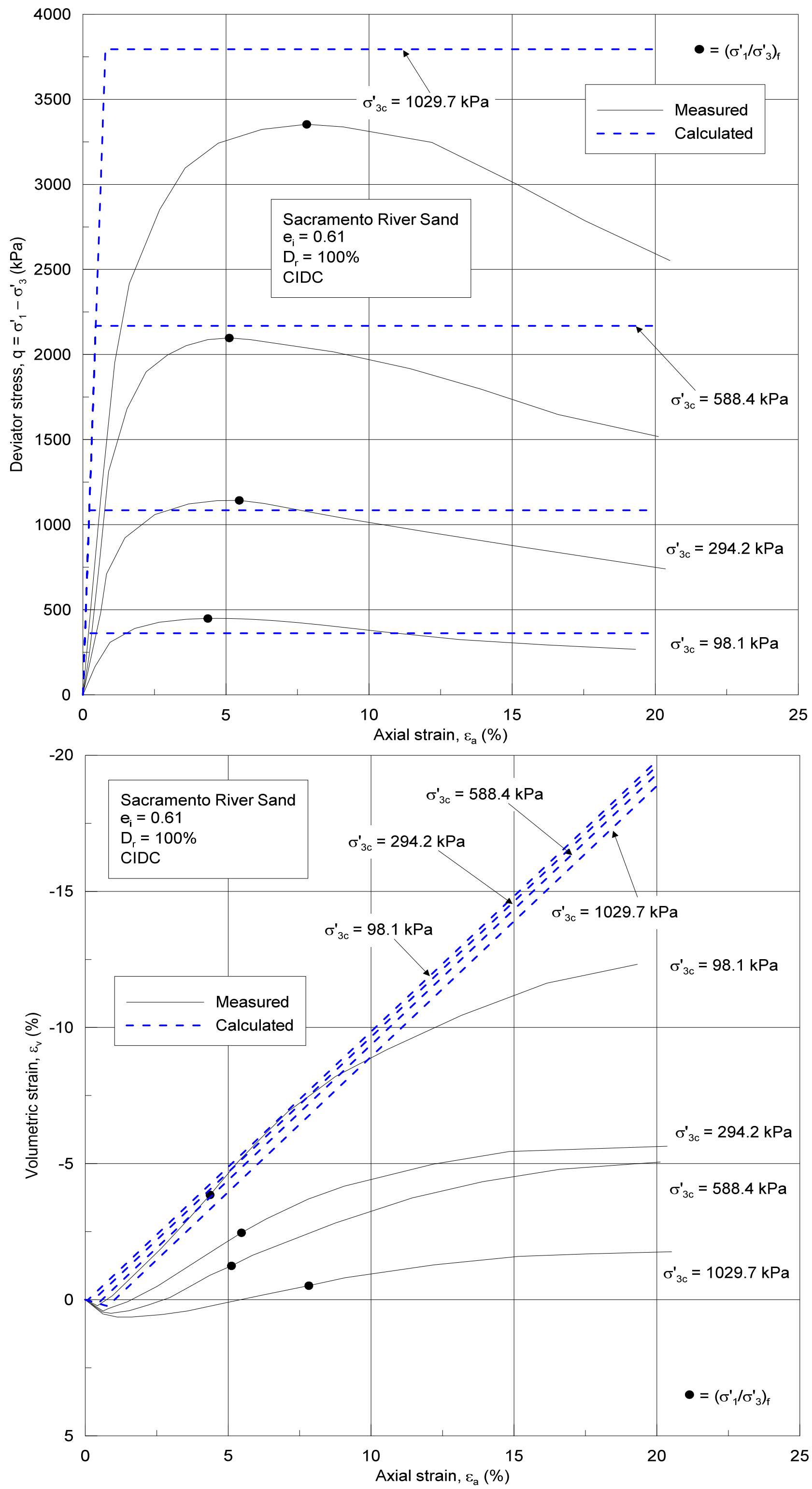


Figure 5.3 – CIDC stress-strain curves of Sacramento River Sand at D_r = 100% with dilation angle specified

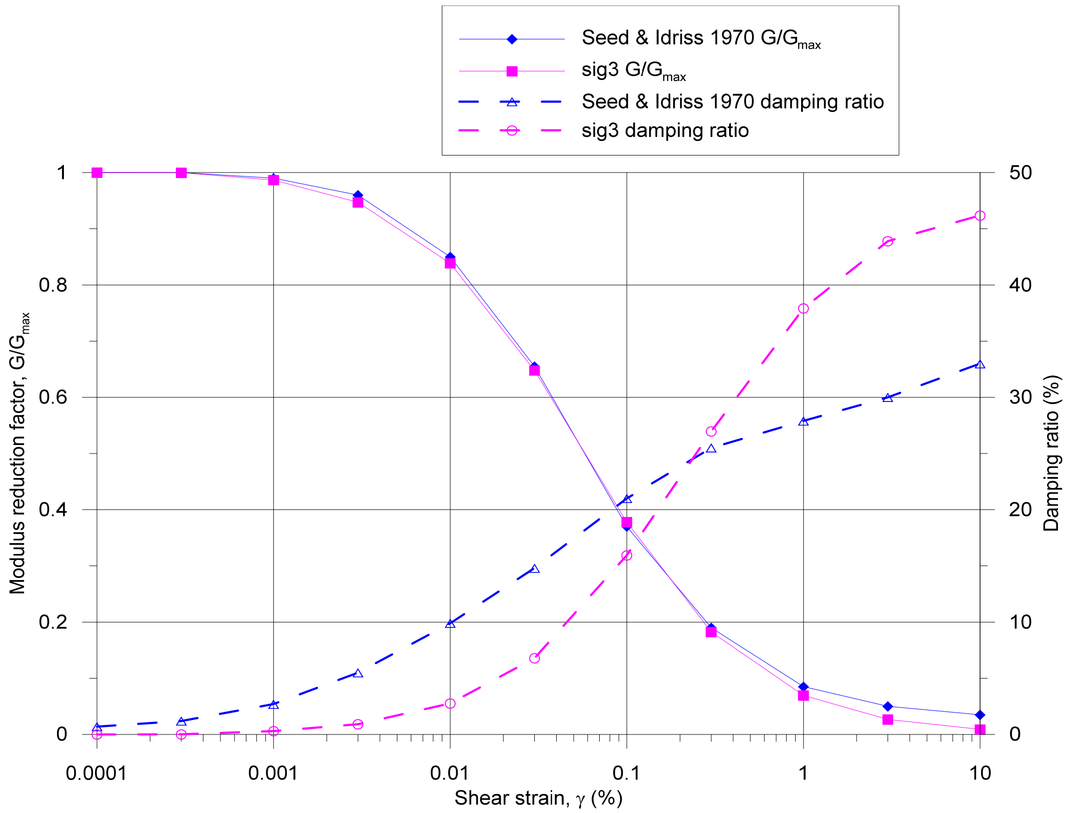


Figure 5.4 – Comparison of modulus reduction and damping ratio curves

5.2.1 Effects of Hysteretic Damping

CIDC tests of Sacramento River Sand with D_r of 100% were simulated using the FLAC Mohr-Coulomb model with hysteretic damping option activated. The stress-strain curves of the CIDC tests are compared in Figure 5.5. In addition, stress-strain curves for the Mohr-Coulomb model with and without hysteretic damping activated are compared in Figure 5.6. Effects of hysteretic damping include:

- Extremely soft and unrealistic responses are observed at axial strain larger than about 0.45%.
- The effect of confining stress (or overburden stress dependency) is not captured by the hysteretic damping option when the confining stress is larger than about 98.1 kPa (2049 lb/ft^2). In other words, friction angle becomes inconsequential at higher confining stress.
- Elastic moduli are significantly reduced with hysteretic damping option when confining stress is larger than about 98.1 kPa.
- Failure or plastic flow can only occur when the confining stress is less than about 98.1 kPa.
- According to the modulus reduction factor recorded (see Figure 5.7), the hysteretic damping option was switched off during plastic flow as observed for σ'_{3c} = 98.1 kPa.

Although nonlinear behavior is captured by the hysteretic damping option, it is only applicable for strain levels less than approximately 0.45%. Hysteretic damping reduces the modulus according to the modulus reduction data specified (i.e., G/G_{max} curve). Due to the reduced modulus, the response could not reach the strength governed by the failure envelope and would result in excessive and unrealistic deformation. Based on the results, hysteretic damping is deemed not suitable for material that could yield during the analysis.

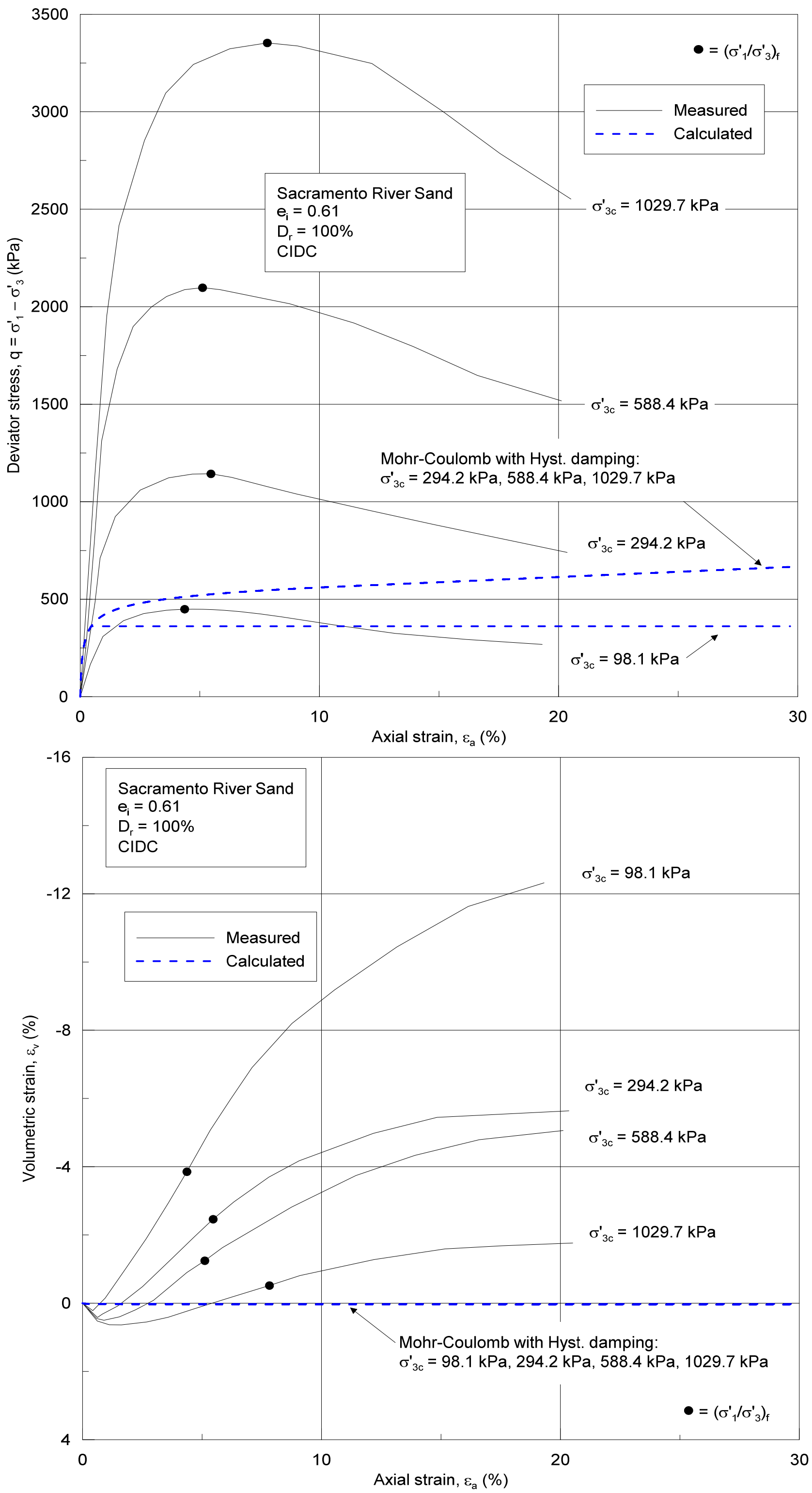


Figure 5.5 – CIDC stress-strain curves of Sacramento River Sand at D_r = 100% with hysteretic damping activated

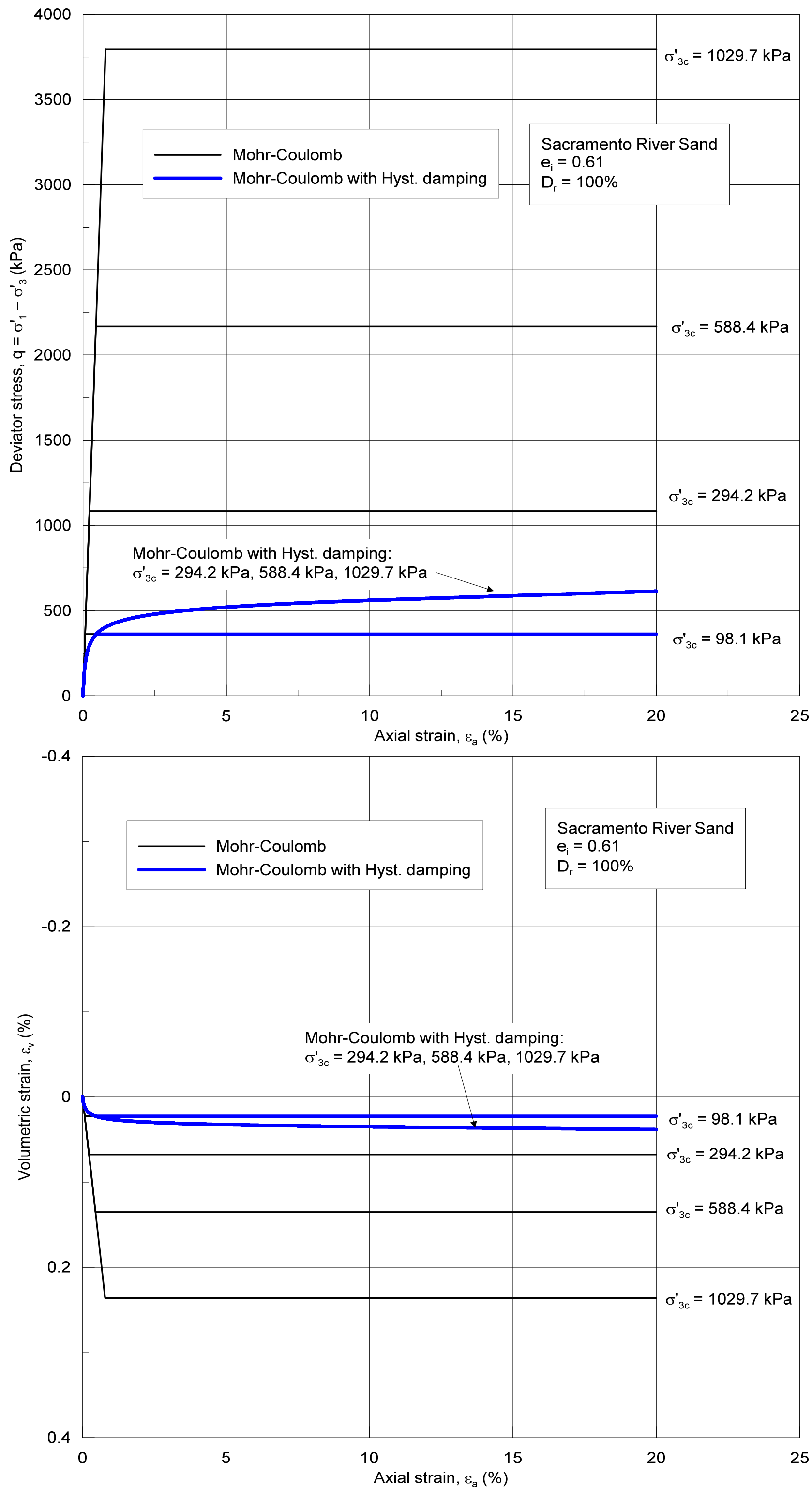


Figure 5.6 – Comparison between Mohr-Coulomb model with and without hysteretic damping option activated

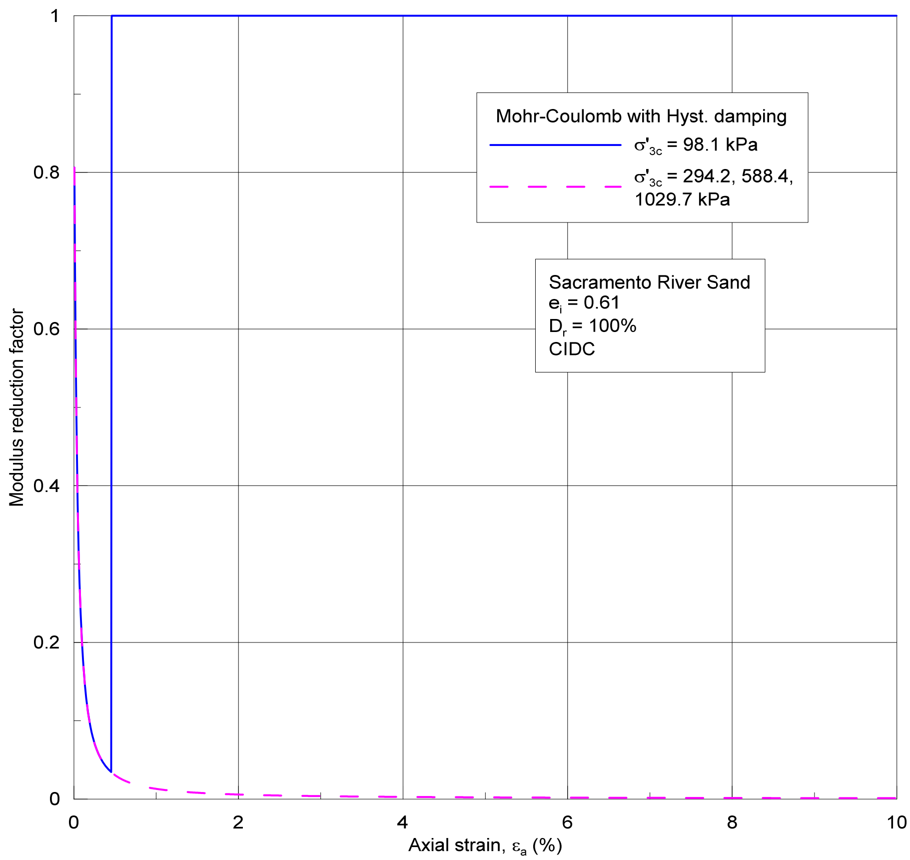


Figure 5.7 – Variation of modulus reduction factor with axial strain

6. Conclusions

Conclusions drawn from the results of the verification assessment include:

1. Nonlinear soil behavior is not captured by the FLAC Mohr-Coulomb model, rather an elastic-perfectly plastic response is generated by the FLAC Mohr-Coulomb model.

2. Strain softening observed in densely packed sand is not captured by the FLAC Mohr-Coulomb model.

3. Failure predicted by the FLAC Mohr-Coulomb model occurs at a much lower strain than the measured response. The calculated failure at low strain level is primarily due to the high shear and bulk moduli specified. The shear and bulk moduli were estimated based on geophysical tests, which are low strain level tests. A way to achieve a better match with the measured behavior might be to lower the values of shear and bulk moduli.

4. When a zero dilation angle is specified in the FLAC Mohr-Coulomb model, volumetric expansion is not generated. However, with a nonzero

dilation angle specified, dilation occurs after failure takes place, and the rate of dilation is constant. In any case, the calculated volumetric strains are not comparable to the measured responses.

5. Current Reclamation FLAC practice is to assign one friction angle to a geologic unit or an embankment zone, which follows the assumption of a linear failure envelope. However, the failure envelopes for most soils are curved. As suggested by Duncan et al. [30], the values of ϕ' decrease in proportion with the logarithm of the confining pressure, and the variation may be represented by:

$$\phi' = \phi_o - \Delta\phi \log_{10}\left(\frac{\sigma'_3}{p_a}\right) \tag{6.1}$$

where ϕ' = secant effective friction angle, ϕ_o = value of ϕ' for σ'_3 = 1 atm, $\Delta\phi$ = reduction in ϕ' for a 10-fold increase in confining pressure, σ'_3 = confining pressure, and p_a = atmospheric pressure. Representative values of ϕ_o and $\Delta\phi$ for various soils are given in Duncan et al. [30]. Equation 6.1 can be implemented in a FLAC embankment dam model using a FISH routine.

6. The calculated volumetric strain remains zero when a constant mean effective stress (p') is specified, which does not agree with the measured behavior.

7. Significant differences between the measured and the calculated responses are observed in the DSS constant-height tests. The FLAC Mohr-Coulomb model fails to generate shear stresses and vertical stresses comparable to the measured DSS constant-height test results.

8. Hysteretic damping drastically reduces the stiffness of the soil when axial strain exceeds approximately 0.45% in a CIDC test. The reduced stiffness is so low that the Mohr-Coulomb strength could not be attained, which is considered erroneous. Caution must be exercised when specifying hysteretic damping. Use of hysteretic damping is not appropriate if yielding of the material is expected during the deformation analysis.

9. The impact of using a primitive model, such as the FLAC Mohr-Coulomb model, on the prediction of embankment deformation is not clear. However, from the calculated stress-strain responses, it is anticipated that the Mohr-Coulomb model would result in a brittle failure (i.e., small strain failure) rather than a ductile failure (i.e., large strain failure).

7. RECOMMENDATIONS FOR FUTURE STUDIES

Recommendations for the future V&V studies include:

1. Perform a single-element model test for other monotonic loading conditions, such as plane strain compression/extension tests and torsional shear tests.

2. Use Reclamation laboratory apparatuses to perform additional shear strength tests for granular materials under different stress paths and drainage conditions. This will provide original test data, including artifacts and anomalies, to the Reclamation analysts that may not otherwise be available in the literature.

3. Evaluate the effectiveness of other FLAC built-in constitutive models by comparion with laboratory stress-strain data presented in this study.

4. Evaluate the effects of soil dilation angle on the seismic performance of an embankment dam model.

5. Current Reclamation practice is to perform non-coupled or mechanical only dynamic analysis. However, excessive pore water pressure generation is closely related to the triggering of soil liquefaction. Future studies should include single-element undrained modeling of granular material under both monotonic and dynamic loadings. This will provide insights to the capabilities of FLAC built-in constitutive models subjected to undrained loading.

6. As part of the validation exercise, compare FLAC simulation against centrifuge model tests of embankment dam under static and seismic loading. Physical model test such as centrifuge test will provide known boundary conditions and material behavior, which are critical to a meaningful and successful numerical simulation. The validation exercise will provide a level of inference, and hence the confidence, in the calculated results of an embankment dam utilizing the current Reclamation simulation procedure.

7. At Reclamation, FLAC has been used primarily in predicting deformations of embankment dams. Concrete dams, however, have been analyzed by the finite element program LS-DYNA. Although not part of a validation exercise, a code-to-code comparison (i.e., FLAC versus LS-DYNA) for embankment dams might reveal advantages and disadvantages of either code in terms of numerical instabilities, computation time, etc.

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ACKNOWLEDGEMENTS

This study is sponsored by the Bureau of Reclamation Dam Safety Technical Development Program. The digital data of Nevada Sand provided by Professor Kanthasamy Muraleetharan of University of Oklahoma, Professor Bruce Kutter of University of California, Davis, and Professor Yie-Ruey Chen of Chang Jung Christian University are gratefully acknowledged.

APPENDIX

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```
;Sacramento River Sand, HC, Dr = 38% (Fig. 4.6)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.8984 bulk 261812.3 shear 87270.8
prop fric 34.0

; --- BC ---
fix x
fix y
ini sxx 0.0 syy 0.0 szz 0.0
ini xvel -1e-6 i 2
ini yvel -1e-6 j 2

; --- Histories ---
his nstep 50
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Loading-Unloading-Reloading ---
;def trip
;  loop i (i,3)
;    command
;    ini xvel -1e-6 i 2
;    ini yvel -1e-6 j 2
;    step 300
;    ini xvel mul -0.1 yvel mul -0.1
;    step 3000
;    ini xvel mul -1.0 yvel mul -1.0
;    step 3000
;    end_command
;  end_loop
;end

; --- Model Solution ---
;step 1000
;trip
step 100000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
```

```
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Sacramento River Sand, HC, Dr = 100% (Fig. 4.6)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.04348 bulk 535185.5 shear 178395.2
prop fric 40.4

; --- BC ---
fix x
fix y
ini sxx 0.0 syy 0.0 szz 0.0
ini xvel -1e-6 i 2
ini yvel -1e-6 j 2

; --- Histories ---
his nstep 50
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Loading-Unloading-Reloading ---
;def trip
;  loop i (i,3)
;    command
;    ini xvel -1e-6 i 2
;    ini yvel -1e-6 j 2
;    step 300
;    ini xvel mul -0.1 yvel mul -0.1
;    step 3000
;    ini xvel mul -1.0 yvel mul -1.0
;    step 3000
;    end_command
;  end_loop
;end

; --- Model Solution ---
;step 1000
;trip
step 100000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
```

```
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Sacramento River Sand, CIDC, Dr = 38% (Fig. 4.7)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.89840 bulk 261812.3 shear 87270.8
prop fric 34.0

; --- BC ---
fix y

;confining stress = 98.1 kPa
ini sxx -98.1 syy -98.1 szz -98.1
apply sxx -98.1 i 2

;confining stress = 196.1 kPa
;ini sxx -196.1 syy -196.1 szz -196.1
;apply sxx -196.1 i 2

;confining stress = 441.3 kPa
;ini sxx -441.3 syy -441.3 szz -441.3
;apply sxx -441.3 i 2

;confining stress = 1245.4 kPa
;ini sxx -1245.4 syy -1245.4 szz -1245.4
;apply sxx -1245.4 i 2

ini yvel -1e-6 j 2

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
```

```
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Sacramento River Sand, CIDC, Dr = 100% (Fig. 4.8)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.04348 bulk 535185.5 shear 178395.2
prop fric 40.4

; --- BC ---
fix y

;confining stress = 98.1 kPa
ini sxx -98.1 syy -98.1 szz -98.1
apply sxx -98.1 i 2

;confining stress = 294.2 kPa
;ini sxx -294.2 syy -294.2 szz -294.2
;apply sxx -294.2 i 2

;confining stress = 588.4 kPa
;ini sxx -588.4 syy -588.4 szz -588.4
;apply sxx -588.4 i 2

;confining stress = 1029.7 kPa
;ini sxx -1029.7 syy -1029.7 szz -1029.7
;apply sxx -1029.7 i 2

ini yvel -1e-6 j 2

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
```

```
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Hostun Sand, CIDC, Dr = 65.3% (Fig. 4.14)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96975 bulk 346407.2 shear 115469.1
prop fric 36.0

; --- BC ---
fix y

;confining stress = 200 kPa
ini sxx -200.0 syy -200.0 szz -200.0
apply sxx -200.0 i 2

;confining stress = 500 kPa
;ini sxx -500.0 syy -500.0 szz -500.0
;apply sxx -500.0 i 2

ini yvel -1e-6 j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
```

```
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDE, Dr = 65.3% (Fig. 4.14)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96975 bulk 346407.2 shear 115469.1
prop fric 36.0

; --- BC ---
fix y

;confining stress = 200 kPa
ini sxx -200.0 syy -200.0 szz -200.0
apply sxx -200.0 i 2

;confining stress = 500 kPa
;ini sxx -500.0 syy -500.0 szz -500.0
;apply sxx -500.0 i 2

ini yvel 1e-6 j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
```

```
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDC, Dr = 92.4% (Fig. 4.15)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.03220 bulk 442252.3 shear 147417.4
prop fric 38.1

; --- BC ---
fix y

;confining stress = 203 kPa
ini sxx -203.0 syy -203.0 szz -203.0
apply sxx -203.0 i 2

;confining stress = 350 kPa
;ini sxx -350.0 syy -350.0 szz -350.0
;apply sxx -350.0 i 2

;confining stress = 500 kPa
;ini sxx -500.0 syy -500.0 szz -500.0
;apply sxx -500.0 i 2

ini yvel -1e-6 j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
```

```
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
step 200000


; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDE, Dr = 92.4% (Fig. 4.15)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.03220 bulk 442252.3 shear 147417.4
prop fric 38.1

; --- BC ---
fix y

;confining stress = 203 kPa
ini sxx -203.0 syy -203.0 szz -203.0
apply sxx -203.0 i 2

;confining stress = 500 kPa
;ini sxx -500.0 syy -500.0 szz -500.0
;apply sxx -500.0 i 2

ini yvel 1e-6 j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
```

```
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDC constant p', Dr = 63.7% (Fig. 4.16)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96694 bulk 342144.1 shear 114048.0
prop fric 35.9

; --- Define Variables ---
def setup
  _yvel = -1e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 200 kPa
ini sxx -200.0 syy -200.0 szz -200.0

;p' = 500 kPa
;ini sxx -500.0 syy -500.0 szz -500.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/160.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
```

```
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDE constant p', Dr = 63.7% (Fig. 4.16)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96694 bulk 342144.1 shear 114048.0
prop fric 35.9

; --- Define Variables ---
def setup
  _yvel = 1e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 200 kPa
ini sxx -200.0 syy -200.0 szz -200.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/160.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
```

```
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDC constant p', Dr = 92.7% (Fig. 4.17)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.03284 bulk 527023.1 shear 175674.4
prop fric 40.3

; --- Define Variables ---
def setup
  _yvel = -1e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 203kPa
ini sxx -203.0 syy -203.0 szz -203.0

;p' = 500kPa
;ini sxx -500.0 syy -500.0 szz -500.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/160.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
```

```
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Hostun Sand, CIDE constant p', Dr = 92.7% (Fig. 4.17)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.03284 bulk 527023.1 shear 175674.4
prop fric 40.3

; --- Define Variables ---
def setup
  _yvel = 1e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 203 kPa
ini sxx -203.0 syy -203.0 szz -203.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/160.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
```

```
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Nevada Sand, oedometer test, Dr = 40% (Fig. 4.20)
new
config extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96143 bulk 293517.4 shear 97839.1
prop fric 34.4

; --- Boundary Conditions ---
fix x y
ini yvel -5e-6 j=2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 50
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---

step 10000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
```

```
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Nevada Sand, oedometer test, Dr = 60% (Fig. 4.20)
new
config extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.00542 bulk 465720.4 shear 155240.1
prop fric 39.1

; --- Boundary Conditions ---
fix x y
ini yvel -5e-6 j=2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 50
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---

step 10000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
```

```
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Nevada Sand, CIDC constant p', Dr = 40% (Fig. 4.21)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96143 bulk 293517.4 shear 97839.1
prop fric 34.4

; --- Define Variables ---
def setup
  _yvel = -0.5e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 40 kPa
ini sxx -40.0 syy -40.0 szz -40.0

;p' = 80 kPa
;ini sxx -80.0 syy -80.0 szz -80.0

;p' = 160 kPa
;ini sxx -160.0 syy -160.0 szz -160.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/40.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Model Solution ---
step 400000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
```

```
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Nevada Sand, CIDE constant p', Dr = 40% (Fig. 4.21)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96143 bulk 293517.4 shear 97839.1
prop fric 34.4

; --- Define Variables ---
def setup
  _yvel = 0.5e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 40 kPa
ini sxx -40.0 syy -40.0 szz -40.0

;p' = 80 kPa
;ini sxx -80.0 syy -80.0 szz -80.0

;p' = 160 kPa
;ini sxx -160.0 syy -160.0 szz -160.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/40.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Model Solution ---
step 400000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
```

```
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Nevada Sand, CIDC constant p', Dr = 60% (Fig. 4.22)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.00542 bulk 465720.4 shear 155240.1
prop fric 39.1

; --- Define Variables ---
def setup
  _yvel = -0.5e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 40 kPa
ini sxx -40.0 syy -40.0 szz -40.0

;p' = 80 kPa
;ini sxx -80.0 syy -80.0 szz -80.0

;p' = 160 kPa
;ini sxx -160.0 syy -160.0 szz -160.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/40.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Model Solution ---
step 400000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
```

```
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Nevada Sand, CIDE constant p', Dr = 60% (Fig. 4.22)
new
config axis
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.00542 bulk 465720.4 shear 155240.1
prop fric 39.1

; --- Define Variables ---
def setup
  _yvel = 0.5e-6
;  _gain = 1.0
;  high_vel = 0.6e-6
  _xvel = -1.0*(_yvel/2.0)
end
setup

; --- BC ---

;p' = 40 kPa
ini sxx -40.0 syy -40.0 szz -40.0

;p' = 160 kPa
;ini sxx -160.0 syy -160.0 szz -160.0

fix x y
ini xvel _xvel i 2
ini yvel _yvel j 2

; --- Servo for Constant Mean Stress ---
;def servo_sig0
;  while_stepping
;  _sig = -1*(sxx(1,1)+syy(1,1)+szz(1,1))/3.0
;  _svel = xvel(2,1)-_gain*(1.0-_sig/40.0)
;  if abs(_svel) > high_vel then
;    _svel=sgn(_svel)*high_vel
;  endif
;  xvel(2,1) = _svel
;  xvel(2,2) = _svel
;end

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1

; --- Model Solution ---
step 400000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
```

```
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
```

```
;Nevada Sand, DSS, Dr = 40% (Fig. 4.23)
new
config extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 1.96143 bulk 293517.4 shear 97839.1
prop fric 34.4

; --- Boundary Conditions ---
; lateral earth pressure coefficient, Ko = nu/(1-nu)
; nu = 0.35, Ko = 0.538461538
ini sxx -43.076923 syy -80.0 szz -43.076923
;ini sxx -43.375353 syy -80.554211 szz -43.375353
;ini sxx -228.29133 syy -423.96953 szz -228.29133
fix x y
ini xvel 5e-6 j=2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 50
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---

step 40000

; --- Export History Files ---
```

```
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Nevada Sand, DSS, Dr = 60% (Fig. 4.24)
new
config extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.00542 bulk 465720.4 shear 155240.1
prop fric 39.1

; --- Boundary Conditions ---
; lateral earth pressure coefficient, Ko = nu/(1-nu)
; nu = 0.35, Ko = 0.538461538
ini sxx -43.076923 syy -80.0 szz -43.076923
;ini sxx -43.375353 syy -80.554211 szz -43.375353
;ini sxx -228.29133 syy -423.96953 szz -228.29133
fix x y
ini xvel 5e-6 j=2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 50
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---

step 40000

; --- Export History Files ---
```

```
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Sacramento River Sand, CIDC, Dr = 100%, Dilation angle = 19.4 deg (Fig. 5.3)
new
config axis extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.04348 bulk 535185.5 shear 178395.2
prop fric 40.4 dil 19.4

; --- BC ---
fix y

;confining stress = 98.1 kPa
ini sxx -98.1 syy -98.1 szz -98.1

;confining stress = 294.2 kPa
;ini sxx -294.2 syy -294.2 szz -294.2

;confining stress = 588.4 kPa
;ini sxx -588.4 syy -588.4 szz -588.4

;confining stress = 1029.7 kPa
;ini sxx -1029.7 syy -1029.7 szz -1029.7

apply sxx -98.1 i 2
ini yvel -1e-6 j 2

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 100
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
```

```
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1

; --- Model Solution ---
step 200000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
```

```
;Sacramento River Sand, CIDC, Dr = 100%, Hysteretic damping (Fig. 5.5)
new
config axis dyn extra 8
g 1 1

; --- Model Properties ---
model mohr
prop dens 2.04348 bulk 535185.5 shear 178395.2
prop fric 40.4

; --- BC ---
fix y

;confining stress = 98.1 kPa
ini sxx -98.1 syy -98.1 szz -98.1

;confining stress = 294.2 kPa
;ini sxx -294.2 syy -294.2 szz -294.2

;confining stress = 588.4 kPa
;ini sxx -588.4 syy -588.4 szz -588.4

;confining stress = 1029.7 kPa
;ini sxx -1029.7 syy -1029.7 szz -1029.7

;apply sxx -98.1 i 2
apply pressure 98.1 i 2

;apply sxx -294.2 i 2
;apply pressure 294.2 i 2

;apply sxx -588.4 i 2
;apply pressure 588.4 i 2

;apply sxx -1029.7 i 2
;apply pressure 1029.7 i 2

;ini yvel -1e-2 j 2
ini yvel -1e-5 j 2
;set dydt 1e-4
;set dy_damp rayleigh 1.0 4.225
ini dy_damp hyst sig3 1.014 -0.4792 -1.249

; --- FISH strain measures ---
; fsr(i,j,arr) = full strain rate tensor
; fsi(i,j,arr) = full strain increment tensor
; arr = 1 --> xx
; arr = 2 --> yy
; arr = 3 --> zz
; arr = 4 --> xy
def qqq
  array ar(4) ai(4)
  while_stepping
  dum = fsr(1,1,ar)
  dum = fsi(1,1,ai)
  ex_1(1,1) = ar(1)
  ex_2(1,1) = ar(2)
  ex_3(1,1) = ar(3)
  ex_4(1,1) = ar(4)
  ex_5(1,1) = ai(1)
  ex_6(1,1) = ai(2)
  ex_7(1,1) = ai(3)
  ex_8(1,1) = ai(4)
end
qqq

; --- Histories ---
his nstep 500
his 1 sxx i 1 j 1
his 2 syy i 1 j 1
his 3 szz i 1 j 1
```

```
his 4 xdisp i 2 j 1
his 5 xdisp i 2 j 2
his 6 xdisp i 1 j 2
his 7 ydisp i 2 j 1
his 8 ydisp i 2 j 2
his 9 ydisp i 1 j 2
his 10 vsi i 1 j 1
his 11 sig1 i 1 j 1
his 12 sig2 i 1 j 1
his 13 sxy i 1 j 1
his 14 ex_1 i 1 j 1
his 15 ex_2 i 1 j 1
his 16 ex_3 i 1 j 1
his 17 ex_4 i 1 j 1
his 18 ex_5 i 1 j 1
his 19 ex_6 i 1 j 1
his 20 ex_7 i 1 j 1
his 21 ex_8 i 1 j 1
his 22 hyst modfac i 1 j 1

; --- Model Solution ---
;step 200000
step 2000000

; --- Export History Files ---
set hisfile=01-sxx.his
hist write 1
set hisfile=02-syy.his
hist write 2
set hisfile=03-szz.his
hist write 3
set hisfile=04-xdisp-2-1.his
hist write 4
set hisfile=05-xdisp-2-2.his
hist write 5
set hisfile=06-xdisp-1-2.his
hist write 6
set hisfile=07-ydisp-2-1.his
hist write 7
set hisfile=08-ydisp-2-2.his
hist write 8
set hisfile=09-ydisp-1-2.his
hist write 9
set hisfile=10-vol-strain.his
hist write 10
set hisfile=11-prin-stress-1.his
hist write 11
set hisfile=12-prin-stress-2.his
hist write 12
set hisfile=13-sxy.his
hist write 13
set hisfile=14-fsr-xx.his
hist write 14
set hisfile=15-fsr-yy.his
hist write 15
set hisfile=16-fsr-zz.his
hist write 16
set hisfile=17-fsr-xy.his
hist write 17
set hisfile=18-fsi-xx.his
hist write 18
set hisfile=19-fsi-yy.his
hist write 19
set hisfile=20-fsi-zz.his
hist write 20
set hisfile=21-fsi-xy.his
hist write 21
set hisfile=22-hyst-modfac.his
hist write 22
```