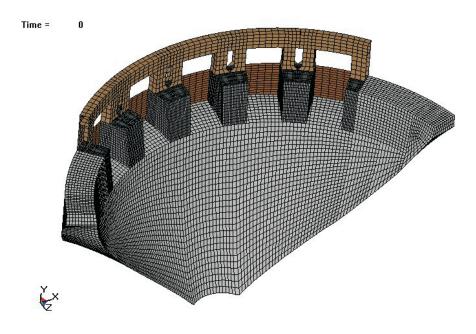
# RECLAMATION Managing Water in the West

Report DSO-06-02

# LS-DYNA vs. DYNA3D Benchmark and Validation Testing



Dam Safety Technology Development Program



U.S. Department of the Interior Bureau of Reclamation Technical Service Center Denver, Colorado

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### BUREAU OF RECLAMATION Technical Service Center, Denver, Colorado Structural Analysis Group, 86-68110

Report DSO-06-02

# LS-DYNA vs. DYNA3D Benchmark and Validation Testing

Dam Safety Technology Development Program Denver, Colorado

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# **Mission Statements**

The mission of the Department of the Interior is to protect and provide access to our Nation's natural and cultural heritage and honor our trust responsibilities to Indian Tribes and our commitments to island communities.

The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.

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Since 2002, the Bureau of Reclamation has used the explicit code DYNA3D, sometimes in conjunction with the implicit code NIKE3D, to evaluate complex three-dimensional finite element models of various dams and buildings. While in some cases the DYNA3D code worked effectively and produced quality results, it was found that in the more complex models where there are tens of thousands of elements and several complex contact surfaces, the model took days to run and its behavior/results were questionable. This led to the consideration of LS-DYNA, both an implicit and explicit code that uses the same basic theory of DYNA3D but is optimized for performance, has improved contact surface interaction, has more hourglass control options, is commercially available, has readily available technical support, and has far superior postprocessing and error checking.

This report summarizes the basic results comparing program performance and stability between DYNA3D and LS-DYNA as well as the validation and benchmark testing of LS-DYNA.

The finite element models (FEM) for Parker Dam and Black Canyon Diversion Dam were selected to be used as the test cases in comparing DYNA3D and LS-DYNA due to their size and contact surface complexity. The load application in this model is as follows:

- Gravity and forces are applied from 0 to 3 seconds.
- The model stabilizes from 3 to 6 seconds.
- The ground motion begins at 6 seconds and runs until 20 seconds.

# **Program Background**

#### DYNA3D

DYNA3D is a general, explicit, finite element program for static and dynamic, linear or nonlinear structural analysis. DYNA3D uses small time steps to integrate the equations of motion to solve transient dynamic problems. An applied force causes movement in the structure. Elements in the model strain at certain rates and resist the load (force = mass x acceleration).

John O. Hallquist developed DYNA3D at the Lawrence Livermore National Laboratory (LLNL). John Hallquist left LLNL in 1987 to establish Livermore

Software Technology Corporation (LSTC) and to commercialize, as LS-DYNA, the public domain code that originated as DYNA3D.

#### LS-DYNA

LS-DYNA has advanced far beyond the original DYNA and, the Structural Analysis Group believes, beyond the present DYNA3D. LS-DYNA incorporates all the LLNL codes (DYNA3D, NIKE3D, Topaz, and ALE3D) into one code. Its fully automated contact analysis capability, a wide range of constitutive models to simulate a whole range of engineering materials (steels, composites, foams, concrete, etc.), error-checking features, and high scalability have enabled users worldwide to successfully solve many complex problems.

As such, LS-DYNA has many features to simulate the physical behavior of 2D and 3D structures: implicit and explicit nonlinear dynamics, thermal, failure, contact, quasistatic, Eulerian, Arbitrary-Lagrangian-Eulerian (ALE), Fluid-Structure-Interaction (FSI) and multiphysics coupling.

LS-DYNA runs on a variety of platforms, including PCs (Windows, Linux), UNIX workstations, supercomputers, and massively parallel processors (MPP). The code is fully vectorized and takes advantage of multiple processors by shared-memory computing (SMP). Parts of a job are distributed to several machines with separate processors and memories. The main advantage of MPP is the better performance in terms of central processing unit (CPU) timing and scalability when many CPUs are used.

# **Program Performance**

#### **Run Time**

The difference in overall program performance (CPU run time) between DYNA3D and LS-DYNA is clear. The code optimization in LS-DYNA resulted in a substantially faster result which not only saves time but permits the user to model with a much finer mesh.

In DYNA3D, the 80,000-element Parker Dam model, which included a 20-second time history ground motion, ran for a total of 5 days. LS-DYNA took only 15 hours to solve the model with 80,000 elements.

Using the same model and ground motion but a much finer mesh of 130,000 elements, the total run time in LS-DYNA was only 24 hours. The mesh density is relevant because the size of the elements directly effects the solution time and

accuracy. The finer the mesh, the longer it takes the program to solve, but it ultimately yields more accurate results. LS-DYNA allows the user to have a denser mesh.

The Structural Analysis Group was forced to create meshes that were too coarse in DYNA3D to shorten run times at the expense of accuracy. The Group believes that finer meshes in LS-DYNA will greatly reduce the run time errors experienced in DYNA3D and still produce high quality results in less time.

#### **Hourglass Control**

Explicit analyses work well for problems with fast loading, nonlinear materials, contact surfaces, wave propagation through materials, fluid elements and large deformations. As such, explicit analysis is uniquely suitable to seismic analysis of dams. The solution is performed by iterating quickly through the elements at very small time steps. To accomplish this, simple eight-node, single integration point elements are used. Mesh densities need to be at least nine times finer than in implicit analysis. Single integration point elements become unstable (hourglass) as the shear in the element increases. To minimize this effect, small elements are used, or numerical hourglass controls are incorporated.

As such, hourglass control is one of the most important factors in explicit analysis. This allows the modeler to more accurately represent the actual terrain, structure, and water with several different mesh densities using contact surfaces as an interface with minimal mathematical error. Hourglassing can be seen when the displaced shape of the model is scaled or in the time history plot for stress in an element

As seen in figure 1, in the scaled, displaced shape of the DYNA3D model, several elements have become mathematically unstable and hourglassed. In figure 3, a plot of the vertical stress with several elements can be seen. The elements selected for figure 3 can also be seen. Hourglassing causes an unrealistic slope in the time history stress plot, meaning that the element is progressively becoming unstable, not by the ground motion, but because of mathematical errors. Engineering judgment must now be used to determine where the influence from the hourglassed elements stops and the good elements/results begin.

A benefit to LS-DYNA is that several different hourglass controls have been developed, giving the user several different options to stabilize models. The effect of an appropriate hourglass control in LS-DYNA can be seen in figure 2, a scaled, displaced shape of the same area that produced problems in DYNA3D. Here the hourglass control managed to minimize the mathematical error across the contact interfaces.

Figure 4 shows an LS-DYNA time history stress plot of elements in the same area as those selected for figure 3. The elements selected for figure 4 can also be seen.

The results are more realistic because hourglassing is controlled. Numerical errors are not propagated throughout the model. LS-DYNA provides model stability and more accurate results.

In addition to the scaled displacement and time history plots, LS-DYNA has the ability to quantify the hourglassing in terms of plotting the Internal Energy and Hourglassing Energy. In doing this, a stable model should always have an Hourglass Energy less than or equal to 10 percent of the Internal Energy. Being able to plot the time history of these energies enables the modeler to quickly evaluate the internal stability and accuracy of the model. The Black Canyon Diversion Dam 2D model has an Hourglass Energy to Internal Energy ratio of less than 8 percent throughout the 20-second run. A time history plot of the Hourglass Energy to Internal Energy ratio can be seen in figure 21.

However, it is noted that when results were taken from the DYNA3D model in areas where hourglassing was not present, the results compared closely to those of LS-DYNA. Because of this, past models and results are still valid.

#### **User Support**

User support from LSTC has been very good. LSTC has a support staff that identified errors in the Parker Dam model and provided an improved mesh. DYNA3D is not a commercial code, and LLNL does not provide this level of support.

### **Preprocessing**

Both LS-DYNA and DYNA3D are fully compatible with the preprocessor, TrueGrid, used by the Structural Analysis Group to create finite element models.

#### **Postprocessor**

LS-DYNA has a far superior postprocessor over DYNA3D. DYNA3D only creates Postscript (.ps) files of results. Inserting these files in Technical Memorandums (TM) is difficult and requires several steps. LS-DYNA creates jpeg files that are easily copied and pasted into TMs. This saves considerable time.

The postprocessor for LS-DYNA can also output animations of deflections, contour plots, and vector plots, something that could not be done with the DYNA3D postprocessor.

The learning curve for this postprocessor is incredibly small as it can be mastered in hours.

#### **Training**

LSTC has a number of classes where as LLNL has minimal training available.

# Validation and Testing

The fundamental behavior across DYNA3D and LS-DYNA is pertinent. In the validation and testing, several areas of the modeling results were compared. The Parker Dam and Black Canyon Diversion Dam models were used for the validation testing. The Parker Dam model is a three-dimensional model that includes the foundation, reservoir, dam, gates, and superstructure. The Black Canyon Diversion Dam model is a two-dimensional model that includes the foundation, reservoir, dam, and ten unbonded lift lines modeled with sliding contacts. Using these two models, the damping, water pressure, stresses, and deflection were validated across the two programs.

#### Parker Dam 3D Model

Parker Dam is a medium-thick concrete arch structure that was constructed between 1939 and 1942. Located on the Colorado River in western Arizona, approximately 14 miles northeast of Parker, Arizona, the dam impounds 646,200 acre-feet of water at reservoir water surface elevation 450.0. The spillway consists of five 50-foot wide by 50-foot high fixed wheel gates symmetrically located at the center of the dam with a spillway crest elevation of 400.0 feet. The spillway gate hoists and controls are located in the control structure above the spillway.

The 3D model was used to compare results between DYNA3D and LS-DYNA. Contact surfaces were used to model the interface between the dam and foundation as well as the dam and superstructure.

#### Static Water Pressure

Checking the modeled static water pressure versus the theoretical value at that same depth is a quick way to check the model initialization. As seen in figures 5 and 6, DYNA3D and LS-DYNA, respectively, initialize within 1 percent of the theoretical water pressure value at the end of the gravity application (3 to 6 seconds). This is imperative because of the need for accurate hydraulic loading before and during the earthquake, which will, in turn, result in more accurate stress results. This also verifies that both DYNA3D and LS-DYNA are calculating water pressure in the same manner.

#### Stress Results

In the validation of stress results between DYNA3D and LS-DYNA, the vertical stresses in the upstream face were compared.

In figure 3, the DYNA3D upstream vertical stress time history plot for elements selected along the upstream face of the dam can be seen. The location of the elements selected for the plot in figure 3 can also be seen. The LS-DYNA upstream vertical stress time history plot for elements selected along the upstream face of the dam can be seen in figure 4. The location of the elements selected for the plot in figure 4 can also be seen.

When comparing the results from the two programs, it can be seen that the stresses initialize within 10 percent of each other; however, the hourglassing in DYNA3D causes an unrealistic slope in the time history stress plot, meaning that the element is progressively becoming unstable, not by the applied ground motion, but because of mathematical errors. Engineering judgment must now be used to determine where the influence from the hourglassed elements stops and the good elements/results begin. The stress time history plot in LS-DYNA shows no slope and is accurate.

#### **Black Canyon Diversion Dam 2D Model**

Black Canyon Diversion Dam is a concrete gravity type diversion built between 1922 and 1924. It is located on the Payette River approximately 5 miles northeast of Emmett, Idaho. The dam has a crest elevation of 2,500 feet, crest length of 1,040 feet, structural height of 183 feet, and hydraulic height of 111.5 feet. Black Canyon Reservoir stores 44,650 acre-feet of water at an elevation of 2497.5 feet and drains an area of 2,680 square miles.

The nonoverflow section model was used to compare results between DYNA3D and LS-DYNA. Contact surfaces were used to model the interface between the dam and foundation as well as the disbonded lift lines within the dam. The nodes on the interface between the dam and water were merged. Core test data were used in calculating resulting static and dynamic coefficients of friction.

#### **Damping**

The foundation and 50,000-year earthquake for Black Canyon Diversion Dam were used to compare the deconvolved earthquake in DYNA3D and LS-DYNA. The Structural Analysis Group uses Rayleigh Mass Damping, which can be used in both programs. The response spectrum of Black Canyon Diversion Dam in DYNA3D and LS-DYNA are shown in figures 7 and 8, respectively. There are few differences between the two, which is expected and validates that both apply damping in the same manner. The only difference is that in LS-DYNA, the modeler defines the mass damping coefficient of Rayleigh Damping,  $\alpha$ , applied to the system globally from time zero, whereas in DYNA3D, the modeler may designate a start and end time of the damping. This time condition has no overall

effect on the results of the model. However, if such a condition arose where timedependent damping were needed, LS-DYNA would have the ability to define the damping based on a load curve.

#### **Static Water Pressure**

As seen in figures 10 and 11, DYNA3D and LS-DYNA, respectively, initialize within 1 percent of the theoretical water pressure value at the end of the gravity application (3 to 6 seconds). This is imperative because of the need for accurate hydraulic loading before and during the earthquake, which will, in turn, result in more accurate stress results. This also verifies that both DYNA3D and LS-DYNA are calculating water pressure in the same manner.

#### **Stress Results**

In the validation of stress results between the DYNA3D and LS-DYNA, the vertical stresses in both the upstream face and downstream face were evaluated.

In figure 16, the DYNA3D upstream vertical stress time history plot in element 3389 can be seen. The location of element 3389 can be seen in figure 15. The LS-DYNA upstream vertical stress time history plot for element 3389 can be seen in figure 17. In comparing the two plots, the overall behavior and frequency of the results compare closely; however, several peaks in the DYNA3D results are almost double the value calculated in LS-DYNA.

In figure 19, the DYNA3D upstream vertical stress time history plot in element 3389 can be seen. The location of element 3389 can be seen in figure 18. The LS-DYNA upstream vertical stress time history plot for element 3389 can be seen in figure 20. In comparing the two plots, the overall behavior and frequency of the results compare to within 1 percent with initial stresses from gravity (3 to 6 seconds). However, again several peaks in the DYNA3D results are almost double the value calculated in LS-DYNA.

The peaks found in the DYNA3D plots of both the upstream and downstream vertical stress plots are nearly double in value in comparison to those of LS-DYNA and are considered erroneous data. LS-DYNA's more accurate stress calculation can be attributed to its improved contact mathematics and interaction.

#### Displacement

The validation of nodal displacement across contacts between DYNA3D and LS-DYNA was done by measuring the total z-direction displacement through the full 20-second earthquake record.

In figure 13, the DYNA3D z-displacement time history plot for the nodes selected in figure 10 can be seen. The LS-DYNA z-displacement time history plot for these same nodes can be seen in figure 14. In comparing the two plots, the overall behavior and frequency of the results compare closely; however, the peak displacements of the nodes are consistently about 20 percent higher in the DYNA3D results versus those of the LS-DYNA model. Considering that the

deconvolved ground motions in the DYNA3D and LS-DYNA models are the same, figures 7 and 8 show that the 20-percent difference between the two programs with regards to the z-displacements is attributed to the improved contact interaction and mathematics across the contacts in LS-DYNA.

# **Conclusions**

Both DYNA3D and LS-DYNA are highly complex programs that produce valuable results for structural analysis issues. Several problem areas have been examined and compared using both codes.

With regard to the total run time of the model, the efficiency and optimization of LS-DYNA are far superior. Its fast solution time has two major benefits. First, debugging and total run time of the model are hours, versus days, which improves the ability of the Structural Analysis Group to pass the savings on to the client. Secondly, the program efficiency permits the modeler to develop a much finer mesh without greatly effecting the total run time, which improves the accuracy of the model.

In terms of the overall solution of the model, both DYNA3D and LS-DYNA use the same basic code and, in the simplest models, produce identical results. In the 2D and 3D models with several complexities introduced, the water pressures, stresses, and displacements could all be validated. However, it is noted that as complex contact surfaces are introduced between parts with different mesh densities or irregular shapes, DYNA3D lacks robust contact surfaces and effective hourglass control, which results in a compromised solution. The total effect of hourglassed elements in DYNA3D could not be measured, but its impact to the results of surrounding elements could be seen, in both the time history plots and scaled displacements. The opposite is true for LS-DYNA, which has several hourglass controls, and their effectiveness can not only be seen in the time history plots and scaled displacements but also quantified and plotted using the program's calculated hourglass energy and internal energy.

# **Figures**

Figure 1.—DYNA3D scaled displacement showing hourglassing (x 1,000).

Figure 2.—LS-DYNA scaled displacement showing hourglassing (x 1,000).

Figure 3.—DYNA3D time history stress plot.

Figure 4.—LS-DYNA time history stress plot.

Figure 5.—DYNA3D Parker Dam water pressure (depth = 144 ft).

Figure 6.—LS-DYNA Parker Dam water pressure (depth = 144 ft).

Figure 7.—DYNA3D Black Canyon Diversion Dam resulting surface motions.

Figure 8.—LS-DYNA Black Canyon Diversion Dam resulting surface motions.

Figure 9.—Element selected for water pressure plots (depth = 56.75 ft).

Figure 10.—DYNA3D Black Canyon Diversion Dam water pressure (depth = 56.75 ft).

Figure 11.—LS-DYNA Black Canyon Diversion Dam water pressure (depth = 56.75 ft).

Figure 12.—Nodes selected for time history of nodal displacement plots.

Figure 13.—DYNA3D time history of nodal displacements for the 50,000-yr loading (in).

Figure 14.—LS-DYNA time history of nodal displacements for the 50,000-yr loading (in).

Figure 15.—Upstream face element selected for vertical stress plots.

Figure 16.—DYNA3D vertical stresses (lb/in<sup>2</sup>) in upstream face of nonoverflow section for the 50,000-year loading.

Figure 17.—LS-DYNA vertical stresses (lb/in²) in upstream face of nonoverflow section for the 50,000-year loading.

Figure 18.—Downstream face elements selected for vertical stress plots.

Figure 19.—DYNA3D vertical stresses (lb/in<sup>2</sup>) in downstream face of nonoverflow section for the 50,000-year loading.

Figure 20.—LS-DYNA vertical stresses (lb/in<sup>2</sup>) in downstream face of nonoverflow section for the 50,000-year loading.

Figure 21.—LS-DYNA internal energy vs. hourglass energy ratio.

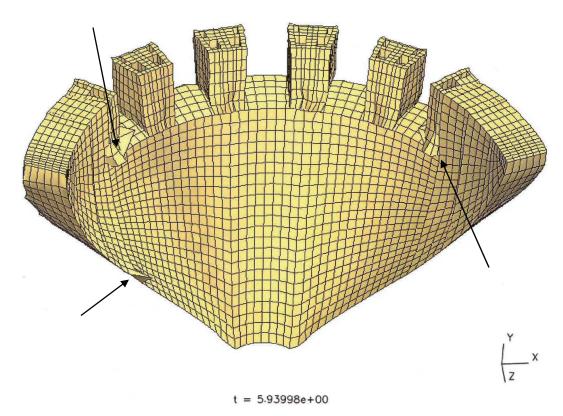
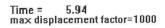


Figure 1.—DYNA3D scaled displacement showing hourglassing (x 1,000).



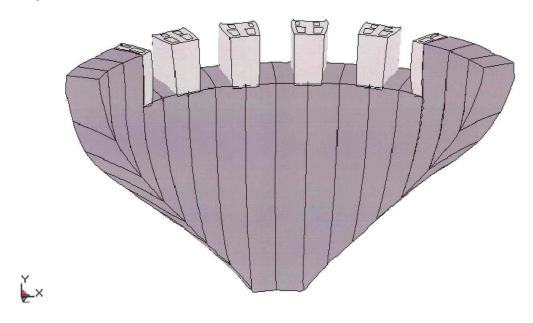
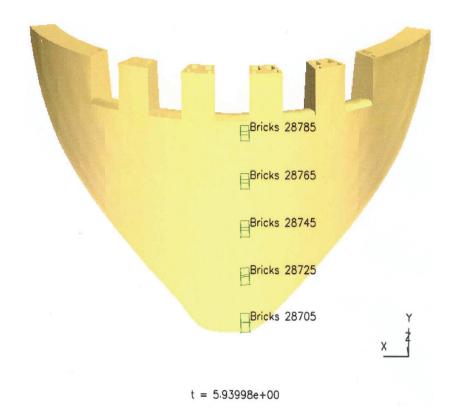


Figure 2.—LS-DYNA scaled displacement showing hourglassing (x 1,000).



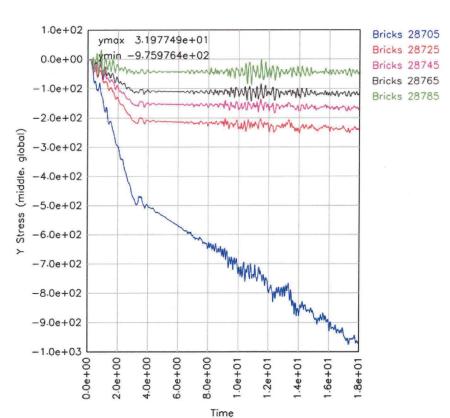


Figure 3.—DYNA3D time history stress plot.

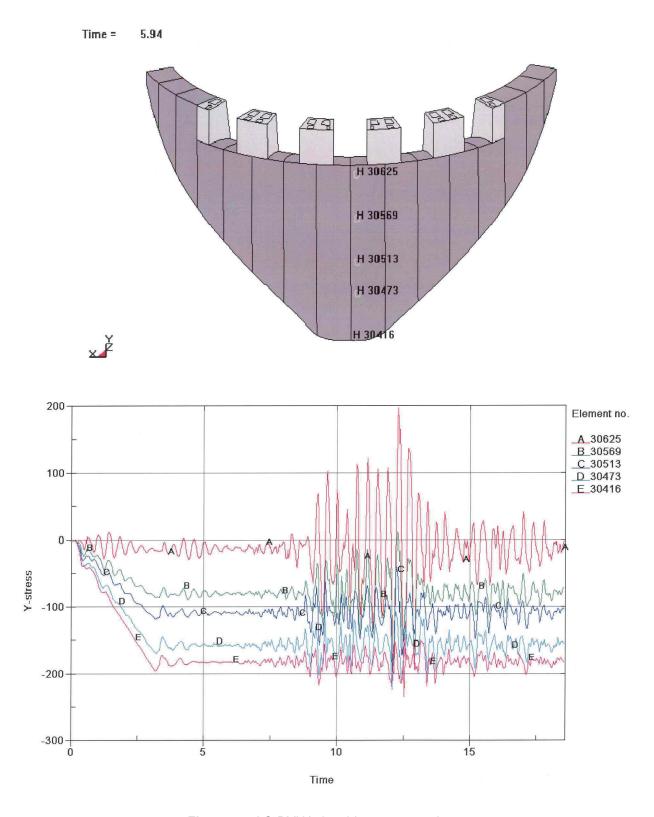
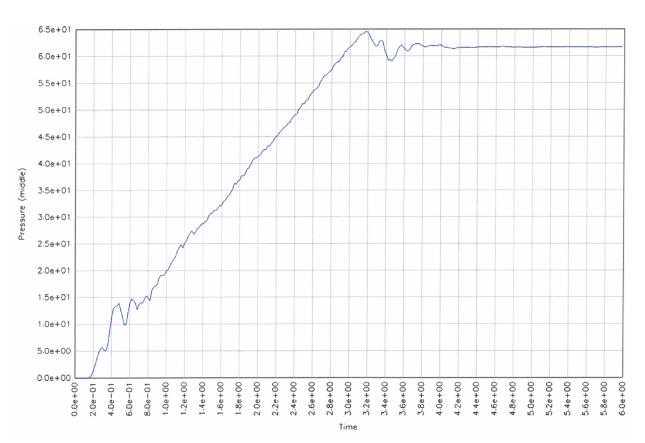


Figure 4.—LS-DYNA time history stress plot.



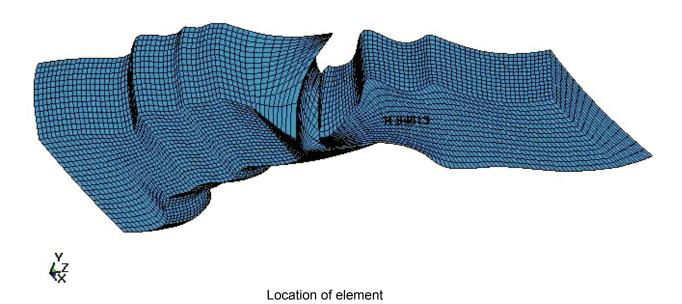
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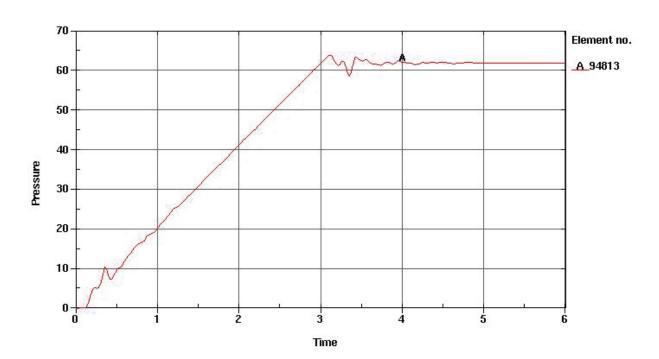


Theoretical static water pressure = 
$$\frac{(144 \text{ ft}) \times (62.4 \text{ lb/ft}^3) \times 1 \text{ ft}}{144 \text{ in}^2/\text{ft}^2} = 62.4 \text{ lb/in}^2$$

Figure 5.—DYNA3D Parker Dam water pressure (depth = 144 ft).

Time = 0





Theoretical static water pressure =  $\frac{(144 \text{ ft}) \times (62.4 \text{ lb/ft}^3) \times 1 \text{ ft}}{144 \text{ in}^2/\text{ft}^2} = 62.4 \text{ lb/in}^2$ 

Figure 6.—LS-DYNA Parker Dam water pressure (depth = 144 ft).

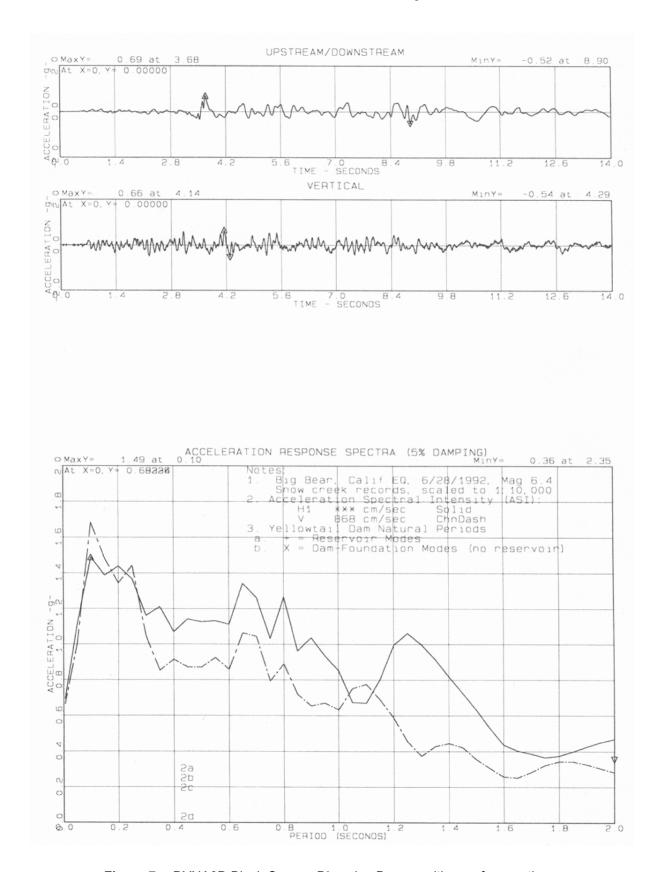
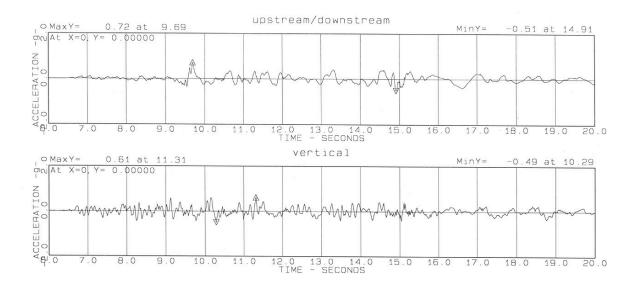


Figure 7.—DYNA3D Black Canyon Diversion Dam resulting surface motions.



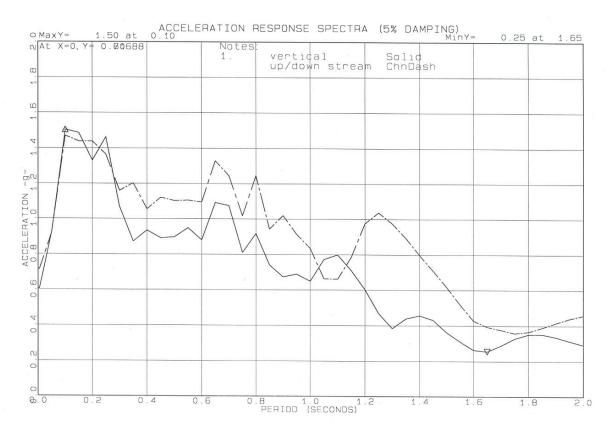


Figure 8.—LS-DYNA Black Canyon Diversion Dam resulting surface motions.

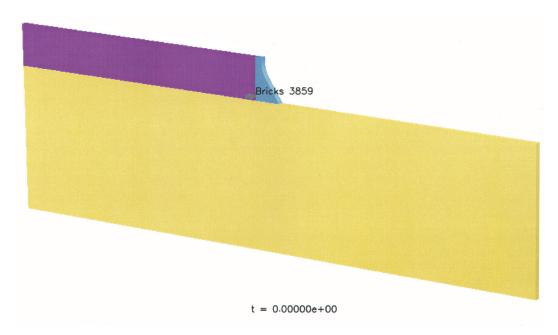
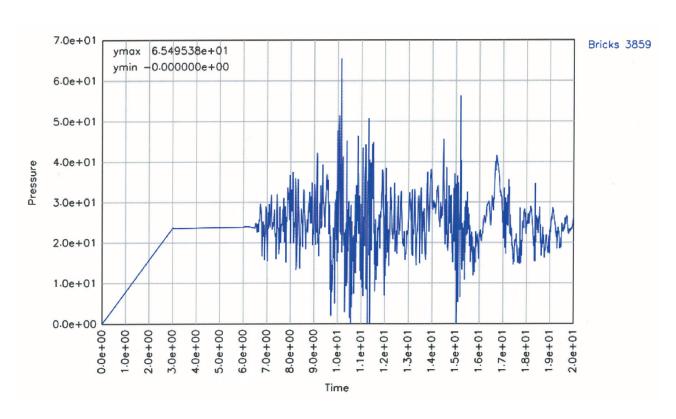
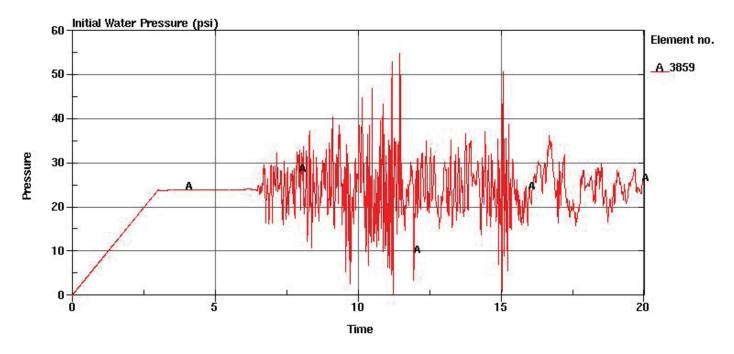


Figure 9.—Element selected for water pressure plots (depth = 56.75 ft).



Theoretical static water pressure =  $\frac{(56.75 \text{ ft}) \times (62.4 \text{ lb/ft}^3) \times 1 \text{ ft}}{144 \text{ in}^2/\text{ft}^2} = 24.59 \text{ lb/in}^2$ 

Figure 10.—DYNA3D Black Canyon Diversion Dam water pressure (depth = 56.75 ft).



Theoretical static water pressure = 
$$\frac{(56.75 \text{ ft}) \times (62.4 \text{ lb/ft}^3) \times 1 \text{ ft}}{144 \text{ in}^2/\text{ft}^2} = 24.59 \text{ lb/in}^2$$

Figure 11.—LS-DYNA Black Canyon Diversion Dam water pressure (depth = 56.75 ft).

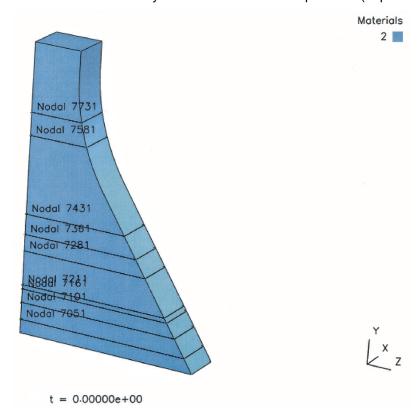


Figure 12.—Nodes selected for time history of nodal displacement plots.

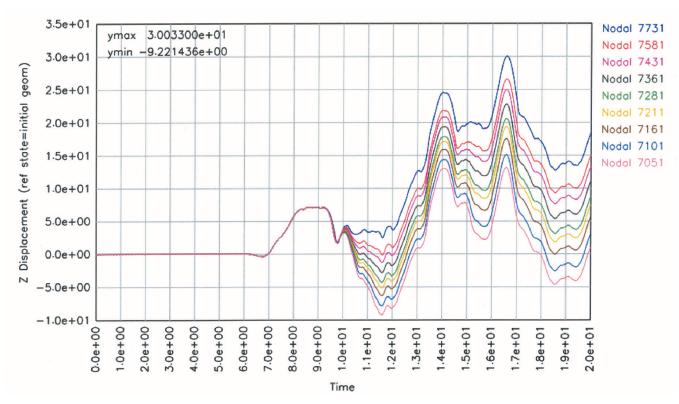


Figure 13.—DYNA3D time history of nodal displacements for the 50,000-yr loading (in).

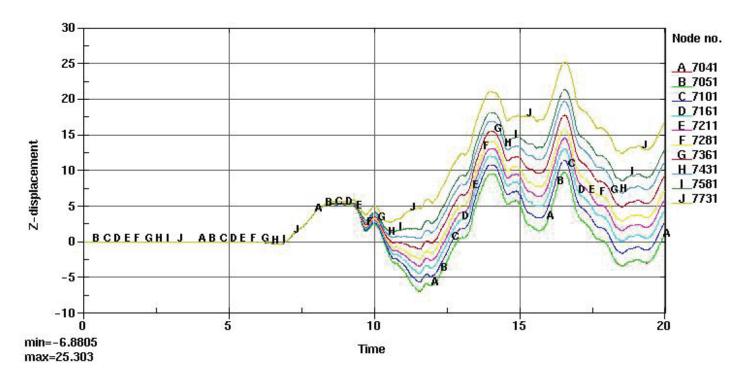


Figure 14.—LS-DYNA time history of nodal displacements for the 50,000-yr loading (in).

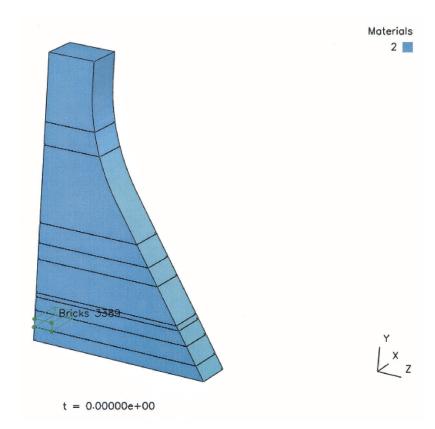
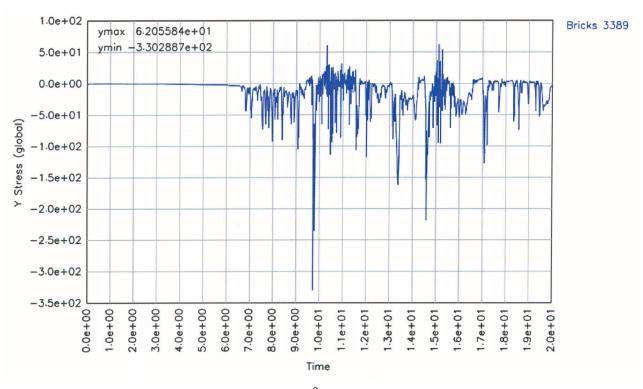
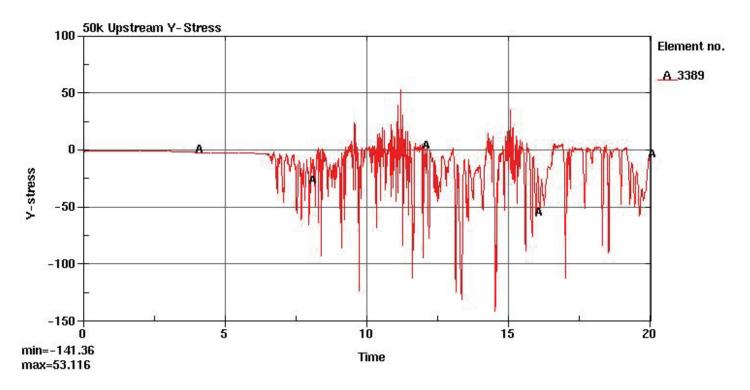


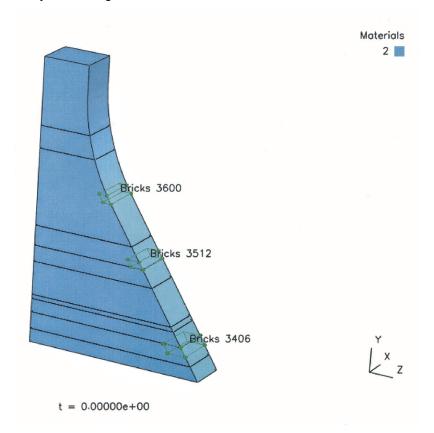
Figure 15.—Upstream face element selected for vertical stress plots.



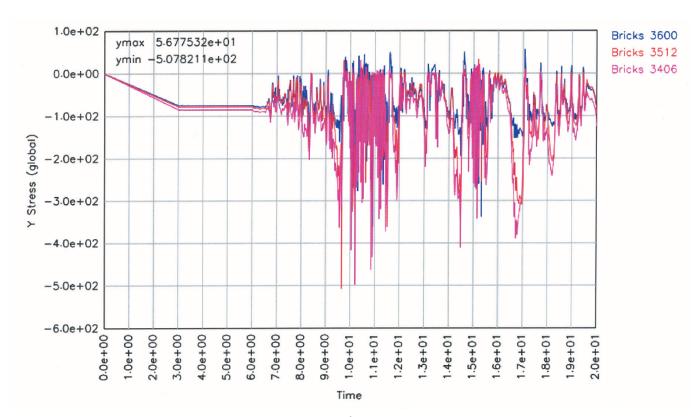
 $\label{eq:figure 16.} \textbf{Eigure 16.} \textbf{--} DYNA3D \ vertical \ stresses \ (lb/in^2) \ in \ upstream \ face \ of \ nonoverflow \ section for the 50,000-year loading.$ 



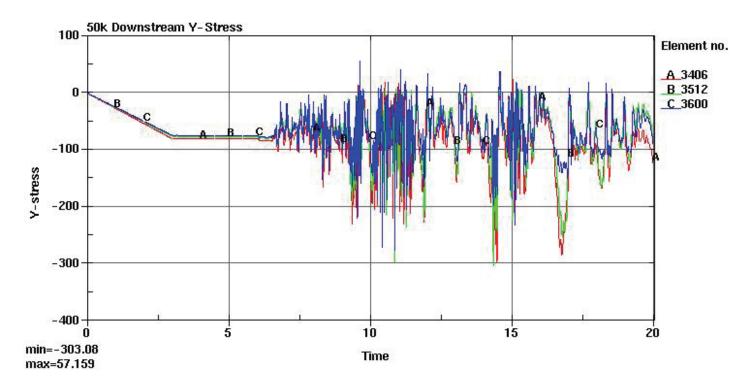
**Figure 17**.—LS-DYNA vertical stresses (lb/in²) in upstream face of nonoverflow section for the 50,000-year loading.



**Figure 18**.—Downstream face elements selected for vertical stress plots.



**Figure 19**.—DYNA3D vertical stresses (lb/in²) in downstream face of nonoverflow section for the 50,000-year loading.



**Figure 20**.—LS-DYNA vertical stresses (lb/in²) in downstream face of nonoverflow section for the 50,000-year loading.

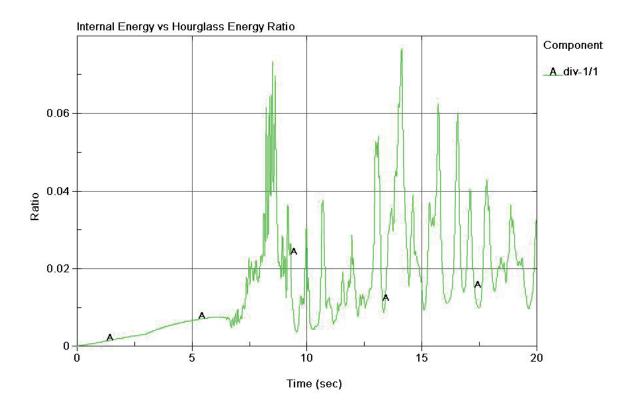


Figure 21.—LS-DYNA internal energy vs. hourglass energy ratio.